

Given below are tables of values for different functions. Classify each function by type. Sketch a graph of the function. Then, state as many specific properties, including the equation if possible, of each function as you possibly can.

1.

| | | | | | | |
|--------|---------------|----------------|----|----|-----------------|----|
| x | -5 | -1 | 0 | 3 | 5 | 9 |
| $F(x)$ | $\frac{1}{3}$ | $-\frac{7}{3}$ | -3 | -5 | $-\frac{19}{3}$ | -9 |

Linear

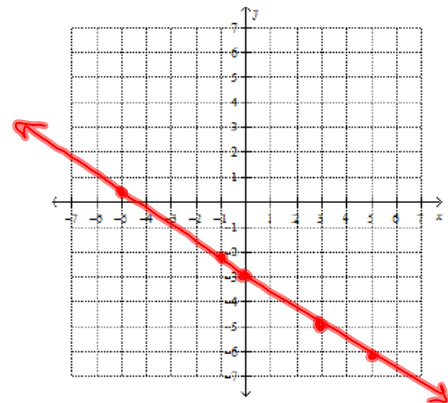
Slope: $-\frac{2}{3}$

y-int: -3

$$y = -\frac{2}{3}x - 3$$

$$D: (-\infty, \infty)$$

$$R: (-\infty, \infty)$$



Jul 25-7:36 AM

2.

| | | | | | | |
|--------|----|----|----|---|---|---|
| x | -6 | -4 | -2 | 0 | 2 | 4 |
| $G(x)$ | 5 | 1 | -3 | 1 | 5 | 9 |

Absolute Value

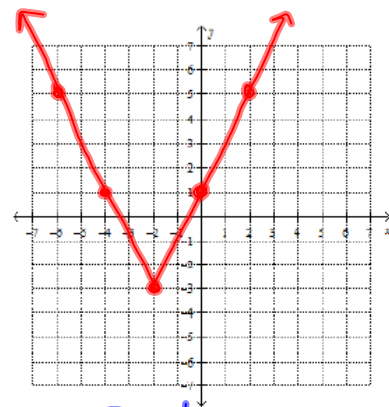
$$y = 2|x + 2| - 3$$

Left 2

Down 3

$$D: (-\infty, \infty)$$

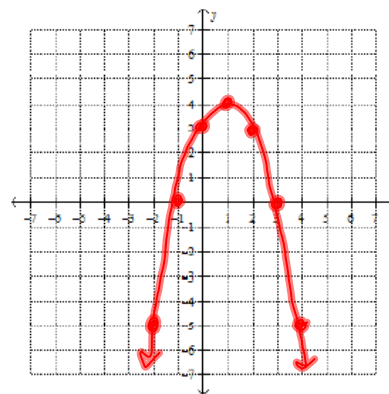
$$R: [-3, \infty)$$



Jul 25-7:33 AM

3.

| | | | | | | | |
|--------|----|----|---|---|---|---|----|
| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| $H(x)$ | -5 | 0 | 3 | 4 | 3 | 0 | -5 |



Quadratic

Zeros: $x = -1$ $x = 3$

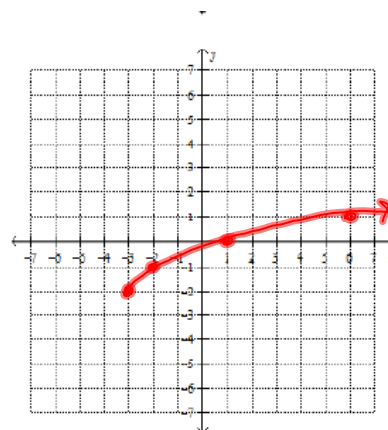
$$y = -(x-1)^2 + 4 \quad \text{or}$$

$$y = -x^2 + 2x + 3$$

Jul 25-7:34 AM

4.

| | | | | | | |
|--------|-----------|----|----|---|---|----|
| x | -4 | -3 | -2 | 1 | 6 | 13 |
| $J(x)$ | Undefined | -2 | -1 | 0 | 1 | 2 |



Square Root
or Radical

$$y = \sqrt{x+3} - 2$$



$$D: [-3, \infty)$$

$$R: [-2, \infty)$$

Jul 25-7:35 AM

5.

| | | | | | | | | |
|--------|--------|-------|-----|---|---|----|----|------|
| x | -6 | -3 | -1 | 0 | 2 | 4 | 6 | 10 |
| $K(x)$ | 3.0156 | 3.125 | 3.5 | 4 | 7 | 19 | 67 | 1027 |

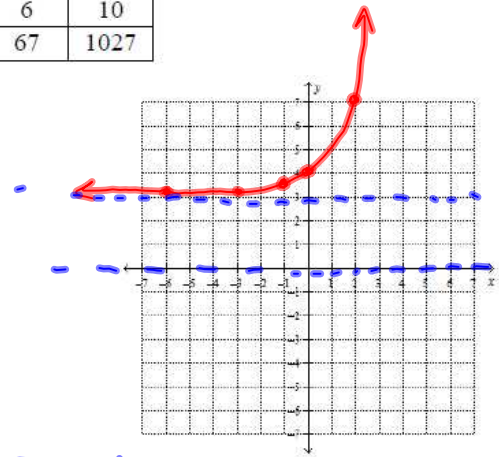
Exponential Growth

2^x
 e^x

$$y = 2^x + 3$$

HA: $y = 3$

$D: (-\infty, \infty)$
 $R: (3, \infty)$



Jul 25-7:36 AM

6.

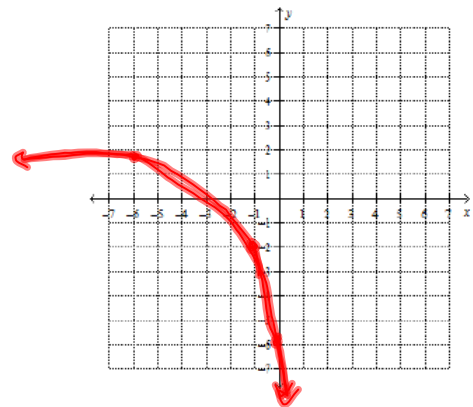
| | | | | | | | | |
|--------|-------|-------|----|----|-----|------|------|-------|
| x | -11 | -6 | -1 | 0 | 2 | 4 | 6 | 10 |
| $M(x)$ | 1.996 | 1.875 | -2 | -6 | -30 | -126 | -510 | -8190 |

Exponential Decay

$$y = -8(2^x) + 2$$

↑
upside
do

↑
y=2



Jul 25-7:36 AM

7.

| | | | | | | | | | |
|--------|--------|--------|-----------|--------|---|-------|-----------|-------|--------|
| x | -1000 | -3.001 | -3 | -2.999 | 0 | 0.999 | 1 | 1.001 | 1000 |
| $N(x)$ | -1.997 | -1.250 | Undefined | -1.249 | 1 | 2998 | Undefined | -3002 | -2.003 |

Rational
 Hole at $x = -3$
 Asymptote at $x = 1$

HA: $y = -2$

$y = \frac{-2(x+1)(x+3)}{(x+3)(x-1)}$
 (Labels: $x+3$ is a Hole, $x-1$ is a vertical asymptote, $y = -2$ is a horizontal asymptote, $x+1$ is the x-intercept)

Jul 25-7:35 AM

$y = \frac{-2(x+1)(x+3)}{(x+3)(x-1)} \rightarrow y = \frac{-2(x+1)}{x-1}$

To find x-int:
 Set $y = 0$

$x-1 \cdot 0 = \frac{-2(x+1)}{x-1}$
 $0 = -2(x+1)$
 $0 = x+1$
 $-1 = x$

Jul 26-8:36 AM

HA

$$D_{\text{Top}} < D_{\text{Bot}}$$

$$\text{HA: } y=0$$

$$D_{\text{Top}} = D_{\text{Bot}}$$

$$\text{HA: } y = \frac{\text{Leading Coefficients}}$$

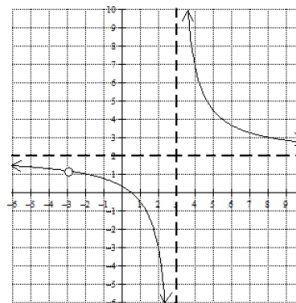
$$D_{\text{Top}} > D_{\text{Bot}}$$

No HA
May have SA

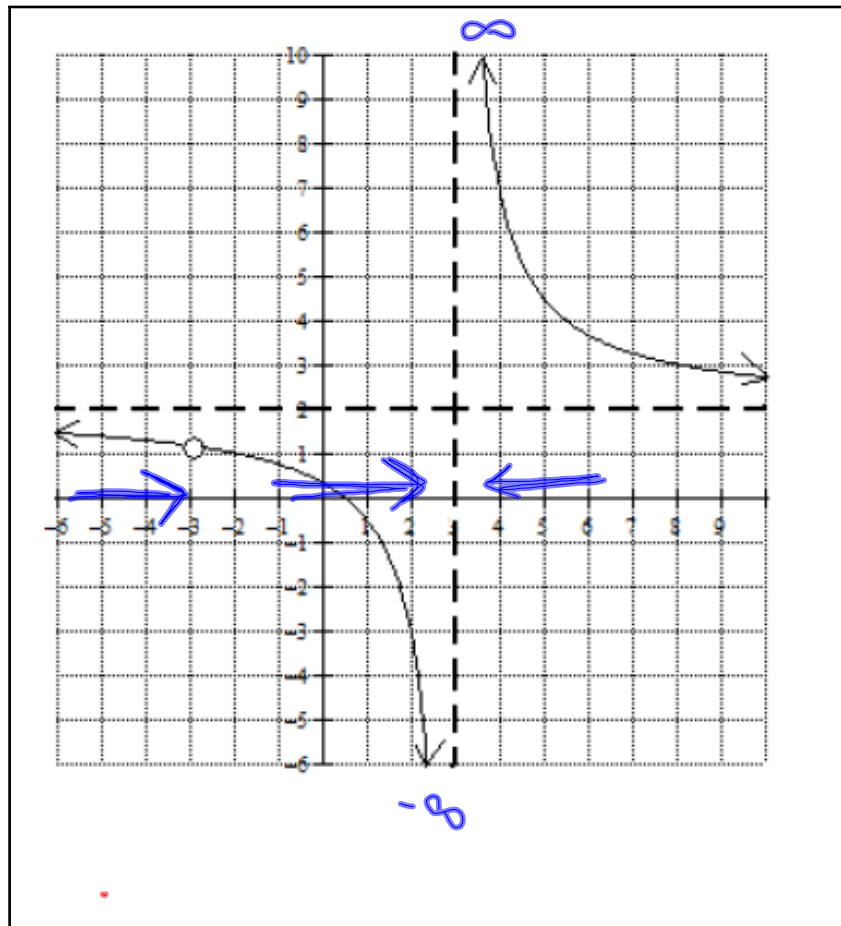
Jul 26-8:41 AM

The equation of the function graphed to the right is

$f(x) = \frac{2x^2 + 5x - 3}{x^2 - 9}$. The coordinates of the hole in the graph are $(-3, \frac{7}{6})$.



Jul 26-7:35 AM



" approaches "

Pre-calculus Statements

As $x \rightarrow -\infty$, the graph of $f(x) \rightarrow$ 2.

As $x \rightarrow \infty$, the graph of $f(x) \rightarrow$ 2.

Calculus Limit Notation

$\lim_{x \rightarrow -\infty} f(x) = 2$

$\lim_{x \rightarrow \infty} f(x) = 2$

Jul 26-7:35 AM

| | |
|---|---|
| As $x \rightarrow -3$ from the left, the graph of $f(x) \rightarrow \underline{7/6}$. | $\lim_{x \rightarrow -3^-} f(x) = 7/6$ |
| As $x \rightarrow -3$ from the right, the graph of $f(x) \rightarrow \underline{7/6}$. | $\lim_{x \rightarrow -3^+} f(x) = 7/6$ |
| As $x \rightarrow 3$ from the left, the graph of $f(x) \rightarrow \underline{-\infty}$. | $\lim_{x \rightarrow 3^-} f(x) = -\infty$ |
| As $x \rightarrow 3$ from the right, the graph of $f(x) \rightarrow \underline{\infty}$. | $\lim_{x \rightarrow 3^+} f(x) = \infty$ |

Jul 26-7:36 AM

Based on what you have just seen, how might you informally define what the value of a limit represents in terms of the graph?

$\lim_{x \rightarrow a} f(x)$ is the y-value
that the graph approaches
as x approaches a .

Jul 26-7:36 AM

A Numerical Analysis of Limits

• Now let's consider the function $f(x) = \frac{(2x^2 + 5x - 3)}{(x^2 - 9)}$

• Complete the table to below to perform a numerical analysis of the function as $x \rightarrow -\infty$ and ∞ .

$-\infty \leftarrow$ $\rightarrow \infty$

| | | | | | | | | |
|------|-------|-------|-------|-------|-------|-------|-------|-------|
| x | -1000 | -500 | -100 | -50 | 50 | 100 | 500 | 1000 |
| f(x) | 1.999 | 1.990 | 1.951 | 1.906 | 2.106 | 2.052 | 2.010 | 2.005 |

$\leftarrow 2$ $\rightarrow 2$

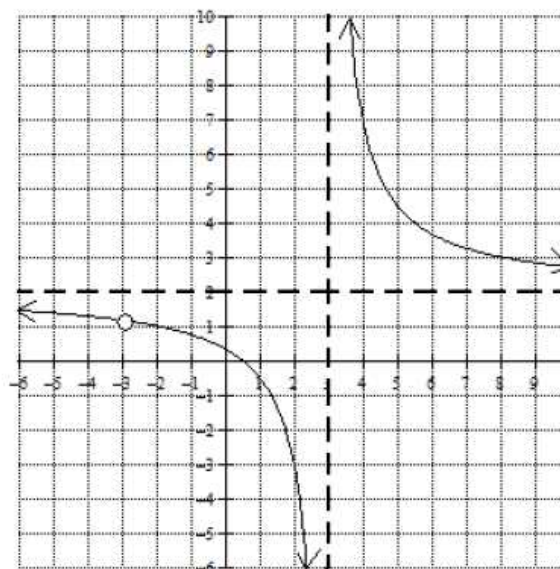
Based on the values in the table, what are the values of

$$\lim_{x \rightarrow -\infty} \frac{2x^2 + 5x - 3}{x^2 - 9} = 2$$

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 5x - 3}{x^2 - 9} = 2$$

Jul 26-7:36 AM

How is the numerical analysis above related in the graph of the function pictured below?



Jul 26-7:37 AM

A Numerical Analysis of Limits

- Now let's consider the function $f(x) = \frac{2x^2 + 5x - 3}{x^2 - 9}$
- Complete the table to below to perform a numerical analysis of the function as $x \rightarrow -3$ from the left and from the right.

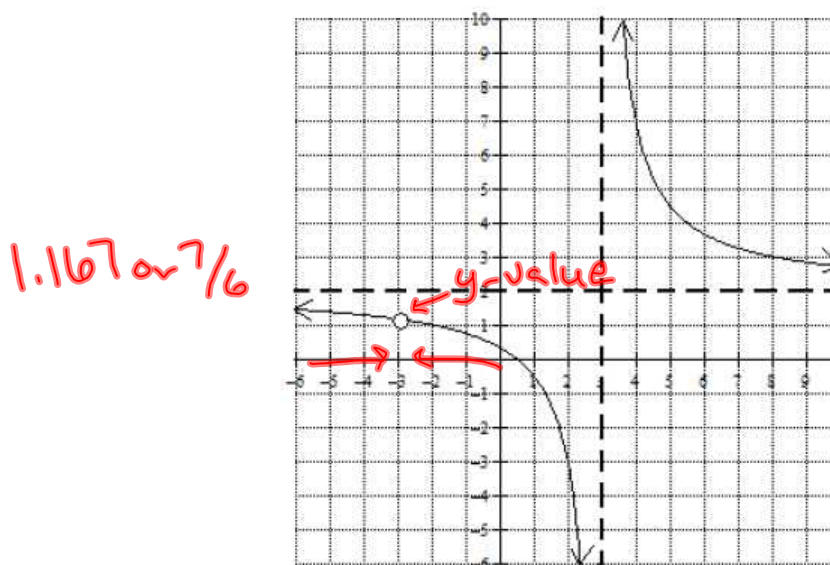
| | | | | | | | | |
|--------|-------|-------|-------|--------|--------|-------|-------|-------|
| x | -3.75 | -3.1 | -3.01 | -3.001 | -2.999 | -2.99 | -2.9 | -2.75 |
| $f(x)$ | 1.259 | 1.180 | 1.168 | 1.167 | 1.167 | 1.165 | 1.153 | 1.130 |

Based on the values in the table, what are the values of

$$\lim_{x \rightarrow -3^-} \frac{2x^2 + 5x - 3}{x^2 - 9} = 1.167 \qquad \lim_{x \rightarrow -3^+} \frac{2x^2 + 5x - 3}{x^2 - 9} = 1.167$$

Jul 27-7:58 AM

How is the numerical analysis above related in the graph of the function pictured below?



Jul 27-7:58 AM

A Numerical Analysis of Limits

- Now let's consider the function $f(x) = \frac{2x^2 + 5x - 3}{x^2 - 9}$
- Complete the table to below to perform a numerical analysis of the function as $x \rightarrow 3$ from the left and from the right.

→ 3 ←

| | | | | | | | | |
|--------|------|-----|------|-------|-------|------|-----|-------|
| x | 2.75 | 2.9 | 2.99 | 2.999 | 3.001 | 3.01 | 3.1 | 3.75 |
| $f(x)$ | -18 | -48 | -498 | -4998 | 5002 | 502 | 52 | 8.667 |

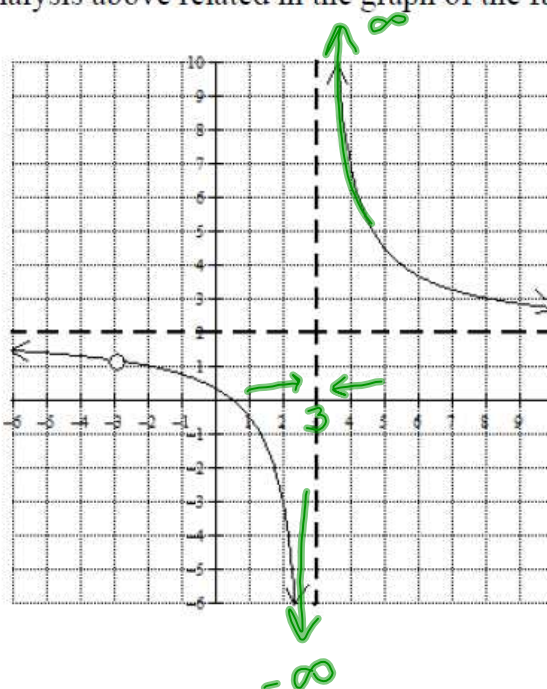
Based on the values in the table, what are the values of

$$\lim_{x \rightarrow 3^-} \frac{2x^2 + 5x - 3}{x^2 - 9} = -\infty$$

$$\lim_{x \rightarrow 3^+} \frac{2x^2 + 5x - 3}{x^2 - 9} = \infty$$

Jul 27-7:58 AM

How is the numerical analysis above related in the graph of the function pictured below?



Jul 27-7:58 AM

Limit Existence Theorem

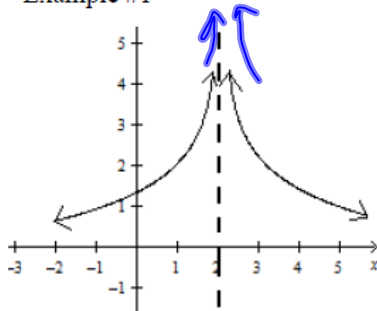
$\lim_{x \rightarrow a} f(x)$ exists iff $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = b$
 where b is any REAL number.

if and
 only if

Jul 28-7:55 AM

Limits That Do Not Exist

Example #1



Find each of the following from the graph.

a) $\lim_{x \rightarrow 2^-} f(x) = \infty$ b) $\lim_{x \rightarrow 2^+} f(x) = \infty$

c) $f(2) = \text{undefined}$

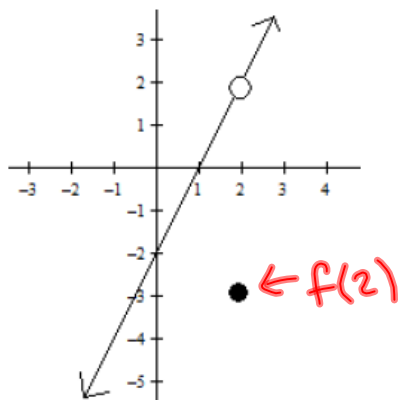
d) Does $\lim_{x \rightarrow 2} f(x)$ exist or not? Why or why not?

$\lim_{x \rightarrow 2} f(x)$ DNE

b/c ∞ is not a REAL
 number

Jul 28-7:56 AM

Example #2



Find each of the following from the graph.

- a) $\lim_{x \rightarrow 2^-} f(x) = 2$
- b) $\lim_{x \rightarrow 2^+} f(x) = 2$
- c) $f(2) = -3$
- d) Does $\lim_{x \rightarrow 2} f(x)$ exist or not? Why or why not?

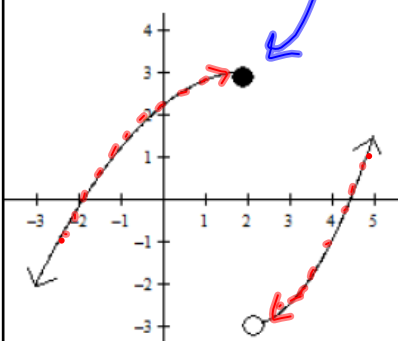
Yes the limit exists

b/c $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$

$$y = \begin{cases} 2x-2 & x \neq 2 \\ -3 & x = 2 \end{cases}$$

Jul 28-7:56 AM

Example #3



Find each of the following from the graph.

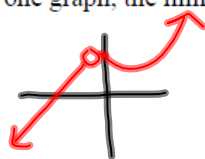
- a) $\lim_{x \rightarrow 2^-} f(x) = 3$
- b) $\lim_{x \rightarrow 2^+} f(x) = -3$
- c) $f(2) = 3$
- d) Does $\lim_{x \rightarrow 2} f(x)$ exist or not? Why or why not?

No, the $\lim_{x \rightarrow 2} f(x)$
DNE b/c

$\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$

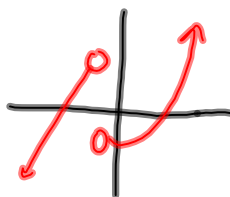
Jul 28-7:56 AM

Based on what you have seen so far, does $f(a)$ have to be defined in order for the $\lim_{x \rightarrow a} f(x)$ to exist? Draw and explain two different graphs to justify your reasoning. In both graphs, $f(a)$ should be undefined but in one graph, the limit should exist while in the second one, it should not exist.



$f(a)$ is undefined but $\lim_{x \rightarrow a}$ still exists.

point discontinuity



$f(a)$ is undef. but $\lim_{x \rightarrow a} f(x)$ DNE.

jump discontinuity

Jul 28-7:57 AM

Below is a table of values of an exponential function. Use the table to find the limits that follow.

| | | | | | | | |
|--------|-----|----|----|----|-----|-------|-------|
| x | -9 | -5 | -3 | -1 | 1 | 3 | 9 |
| $f(x)$ | 513 | 33 | 9 | 3 | 1.5 | 1.125 | 1.002 |

a) $\lim_{x \rightarrow \infty} f(x) = \infty$

b) $\lim_{x \rightarrow -3} f(x) = 9$

c) $\lim_{x \rightarrow 1} f(x) = 1.5$

d) $\lim_{x \rightarrow \infty} f(x) = 1$

Jul 28-7:57 AM

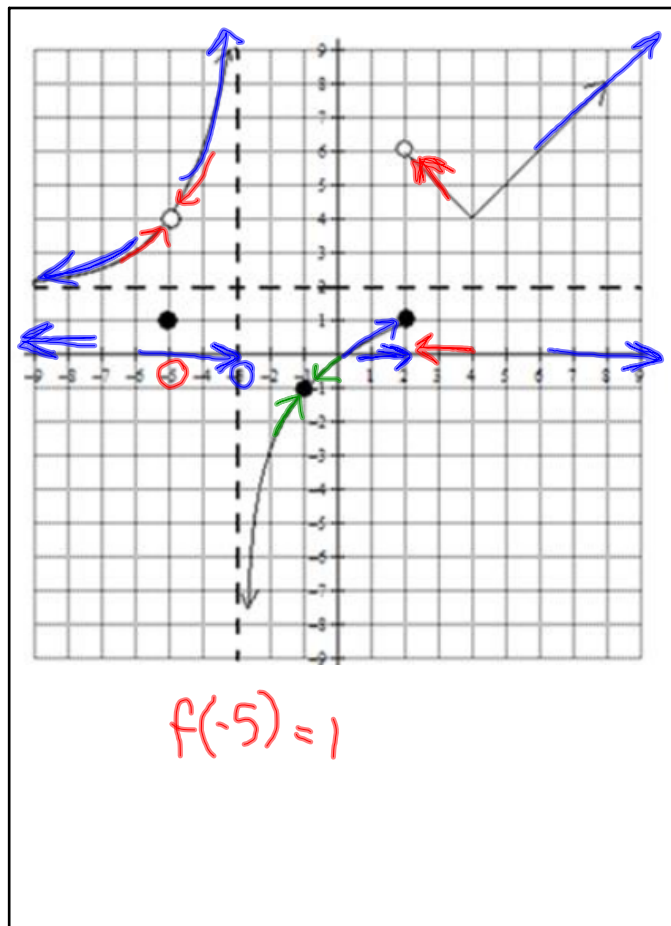
Below is a table of values of a rational function. Use the table to find the limits that follow.

| | | | | | | | | | |
|------|-------|--------|-----------|--------|----|-------|-----------|-------|-------|
| x | -1000 | -1.001 | -1 | -0.999 | 0 | 1.999 | 2 | 2.001 | 1000 |
| F(x) | 1.002 | 2001 | Undefined | -1999 | -1 | 0.333 | Undefined | 0.334 | 0.998 |

a) $\lim_{x \rightarrow -\infty} f(x) = 1$ b) $\lim_{x \rightarrow 1^-} f(x) = \infty$ c) $\lim_{x \rightarrow 1^+} f(x) = -\infty$

d) $\lim_{x \rightarrow 2} f(x) = 1/3$ e) $\lim_{x \rightarrow \infty} f(x) = 1$ f) $\lim_{x \rightarrow -1} f(x) = DNE$

Jul 28-7:57 AM



Jul 28-8:28 AM

Using the graph, find the value of each of the following limits. If a limit does not exist, state why.

a) $\lim_{x \rightarrow -3^-} f(x)$
 ∞

b) $\lim_{x \rightarrow -5} f(x)$
 4

c) $\lim_{x \rightarrow -1} f(x)$
 -1

c) $\lim_{x \rightarrow 3} f(x)$
 DNE

d) $\lim_{x \rightarrow 2^-} f(x)$
 1

e) $\lim_{x \rightarrow 2^+} f(x)$
 6

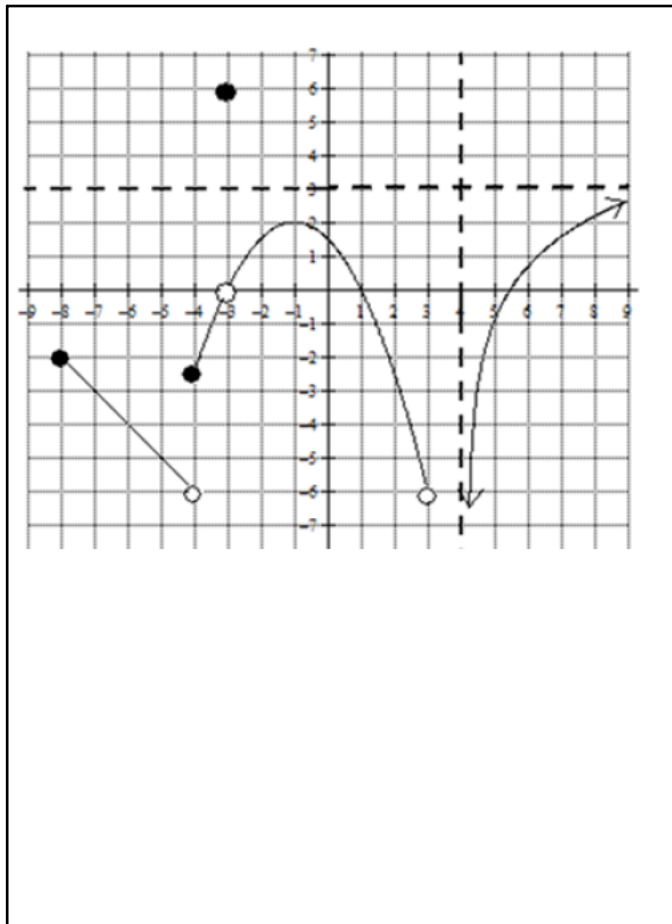
f) $\lim_{x \rightarrow 2} f(x)$
 DNE

g) $\lim_{x \rightarrow -\infty} f(x)$
 2

h) $\lim_{x \rightarrow \infty} f(x)$
 ∞

$\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$

Jul 28-8:28 AM



Jul 28-8:28 AM

a) $\lim_{x \rightarrow 3^-} g(x)$

b) $\lim_{x \rightarrow 6} g(x)$

c) $\lim_{x \rightarrow 1^+} g(x)$

d) $\lim_{x \rightarrow 3^+} g(x)$

e) $\lim_{x \rightarrow 4^-} g(x)$

f) $\lim_{x \rightarrow 4^+} g(x)$

g) $\lim_{x \rightarrow 4} g(x)$

h) $\lim_{x \rightarrow 4^+} g(x)$

i) $\lim_{x \rightarrow 4^-} g(x)$

Completed as Exit Ticket

Jul 28-8:28 AM

Jul 28-8:27 AM