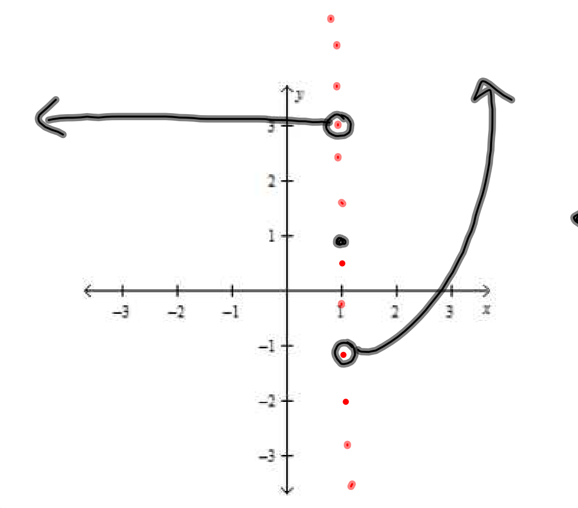
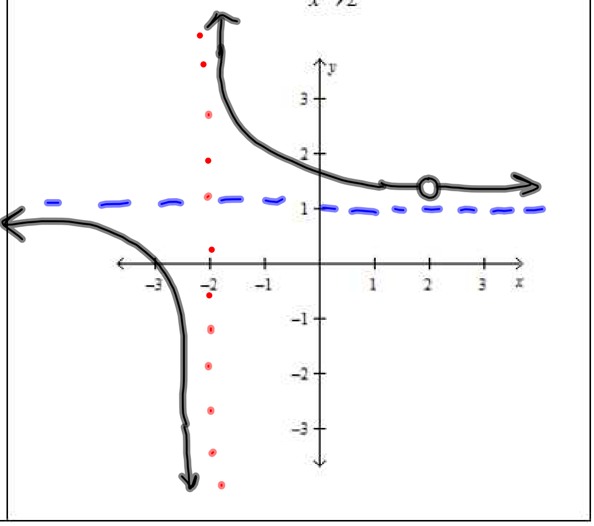
11. $\lim_{x\to 1^-} f(x) = 3$ $\lim_{x\to 1^+} f(x) = -1$ $f(1) = 1$

1



12. $\lim_{x\to -2^-} f(x) = -\infty$ $\lim_{x\to -2^+} f(x) = \infty$

$f(2)$ is undefined but $\lim_{x\to 2} f(x)$ exists.



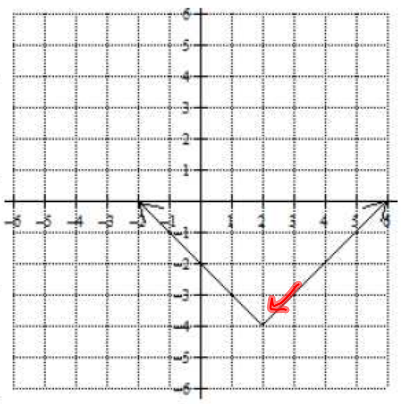
13. In exercise 11, does $\lim_{x\to 1} f(x)$ exist? Explain why or why not.

DNE b/c $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

Jul 31-7:59 AM

Consider the function, $f(x) = \frac{1}{2}\left|-2x+4\right| - 4$, for a moment. The graph of $f(x)$ is pictured below. From the graph, determine the following limits.

$\lim_{x\to a} f(x)$	Find $f(a)$ using the equation.	Find $\lim_{x\to a} f(x)$ from the graph.
$\lim_{x\to 0} f(x)$	$\frac{1}{2}\left|-2(0)+4\right|-4 = -2$	-2
$\lim_{x\to 2^+} f(x)$	$\frac{1}{2}\left|-2(2)+4\right|-4 = -4$	-4
$\lim_{x\to 10} f(x)$	$\frac{1}{2}\left|-2(10)+4\right|-4 = 4$	4



When a function is defined and continuous at a value, $x = a$, how can $\lim_{x\to a} f(x)$ be found analytically?

Provided that $f(x)$ is continuous at $x=a$ then $\lim_{x\to a} f(x) = f(a)$.

Jul 31-8:15 AM

Find each of the following limits analytically. Rational

a) $\lim_{x\to 3} \frac{1}{2}x^2 - 2x + 3$

$\frac{1}{2}(3)^2 - 2(3) + 3$

$= \boxed{1.5}$

b. $\lim_{x\to 3} \frac{5x+2}{2x-3} = \frac{5(3)+2}{2(3)-3}$

$= 5\,2/3 = \boxed{\frac{17}{3}}$

c. $\lim_{x\to 2^-} \frac{\sqrt{x+2}-1}{x+1} = \frac{\sqrt{2+2}-1}{2+1}$

$= \boxed{1/3}$

d. $\lim_{\theta\to\frac{\pi}{2}} \sin 2\theta = \sin\left(\frac{\cancel{2}}{1}\cdot\frac{\pi}{\cancel{2}}\right)$

$= \sin(\pi)$

$= 0$

e. $\lim_{\theta\to\frac{2\pi}{3}} \frac{\cos\theta}{\theta} = \frac{\cos\left(\frac{2\pi}{3}\right)}{\frac{2\pi}{3}}$

$= \frac{-\frac{1}{2}}{\frac{2\pi}{3}} = \frac{-1}{2}\cdot\frac{3}{2\pi}$

$= \boxed{\frac{-3}{4\pi}}$

f. $\lim_{x\to 9} \log_8(11-x)$

$\log_8(11-9)$

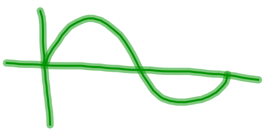
$\log_8 2 = x$

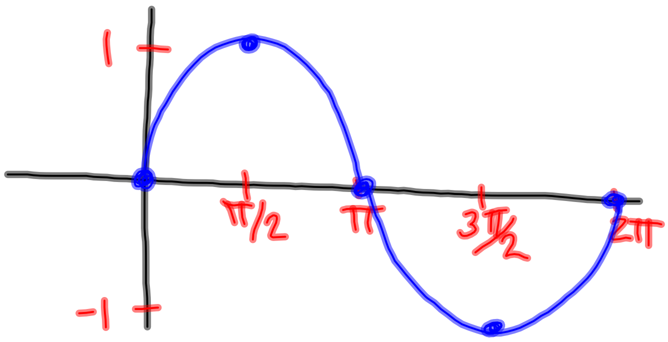
$8^x = 2$

$\boxed{x = 1/3}$

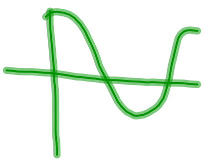
Jul 31-8:15 AM

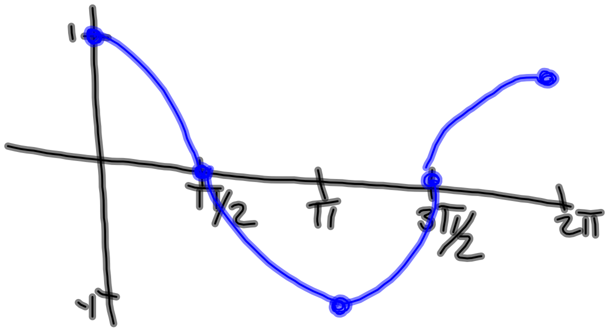
Sin θ





Cos θ





Jul 31-8:35 AM

Analytically Finding Limits of Functions at Undefined Values

What happens if we try to evaluate the limits below by the direct substitution method that was used in the previous six examples?

$$\lim_{x\to -3}\frac{x^2+4x+3}{x^2+2x-3} = \frac{0}{0}$$

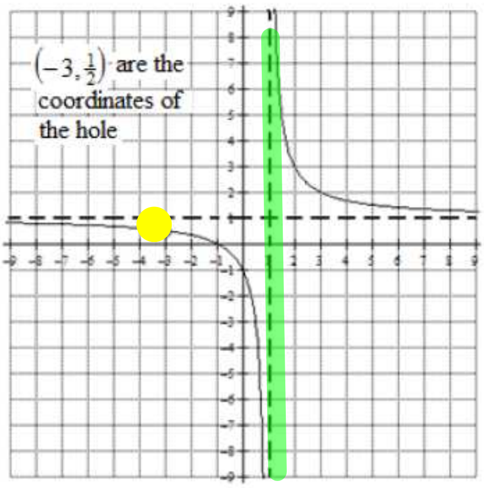
undefined

$$\lim_{x\to 1^-}\frac{x^2+4x+3}{x^2+2x-3} = \frac{8}{0} = \lim_{x\to 1^+}\frac{x^2+4x+3}{x^2+2x-3}$$

undefined

Jul 31-8:49 AM

Just because a function is undefined at a value of x does not mean that a conclusion cannot be reached about the limit. Consider the rational function above. From the graph of the function pictured to the right, what is the value of each limit below?



$$\lim_{x\to -3}\frac{x^2+4x+3}{x^2+2x-3} = \underline{1/2}$$

$$\lim_{x\to 1^-}\frac{x^2+4x+3}{x^2+2x-3} = \underline{-\infty}$$

$$\lim_{x\to 1^+}\frac{x^2+4x+3}{x^2+2x-3} = \underline{\infty}$$

$$\lim_{x\to 1} f(x) = \underline{\text{DNE}}$$

Jul 31-8:51 AM

The task now is to determine how to find these limits analytically. How was it that we found the discontinuities of a rational function in pre-calculus?

① Factor EVERYTHING

② If a factor cancels, the function had a point discontinuity when the factor = 0

③ If a factor in the denominator does not cancel, the function had a nonremovable infinite discontinuity

Jul 31-8:51 AM

We will perform the same algebraic analysis to find the limit of the removable, point discontinuities. Let's do this **Cancellation Process** below.

$$\lim_{x\to -3}\frac{x^2+4x+3}{x^2+2x-3} = \lim_{x\to -3}\frac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-1)}$$

$-3+3$

$$= \lim_{x\to -3}\frac{x+1}{x-1} = \frac{-3+1}{-3-1} = \frac{-2}{-4} = \boxed{\frac{1}{2}}$$

Jul 31-8:51 AM

Based on our knowledge from pre-calculus, we know that if a rational function has a non-removable infinite discontinuity, graphically a Vertical Asymptote exists. Since the y – values do not approach one specific value from both sides at a Vertical Asymptote, then the limit does not exist. However, we can determine if the one sided limits approach $-\infty$ or ∞. In order to do this analytically, we will marry the numerical, graphical, and algebraic approaches. For each limit below, determine the sign of the simplified function at the value to the right or the left of $x = 1$.

Aug 1-8:34 AM

$$\lim_{x\to 1^-} \frac{x^2+4x+3}{x^2+2x-3} = \frac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-1)}$$

$$\lim_{x\to 1^+} \frac{x^2+4x+3}{x^2+2x-3}$$

Value of x to the left of $x = 1$	Simplified function $\frac{x+1}{x-1}$
0.9	$\frac{.9+1}{.9-1} = \frac{+}{-} = \text{Neg}$

Value of x to the right of $x = 1$	Simplified function $\frac{x+1}{x-1}$
1.1	$\frac{1.1+1}{1.1-1} = \frac{+}{+} = \text{Pos}$

$$\lim_{x\to 1^-} \frac{x+1}{x-1} = -\infty$$

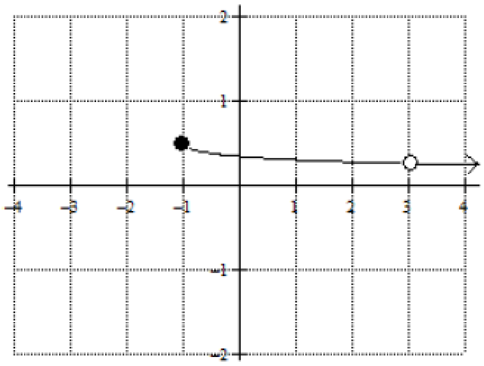
$$\lim_{x\to 1^+} \frac{x+1}{x-1} = \infty$$

Aug 1-8:35 AM

Conjugate

The graph of a function $g(x) = \frac{\sqrt{x+1}-2}{x-3}$ is pictured to the right. Often, **rationalization** can be used to evaluate a limit analytically. Find the following limit.

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$$



$$\lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2} = \boxed{\frac{1}{4}}$$

Aug 1-8:36 AM

$$\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$$

$$(\sqrt{x+1}-2)(\sqrt{x+1}+2)$$

$$x+1 + \cancel{2\sqrt{x+1}} - \cancel{2\sqrt{x+1}} - 4$$

$$x+1-4$$

$$x-3$$

Aug 1-8:45 AM

Find the following Limits Analytically (w/out graphing)

① $\lim\limits_{x\to 3} x^2+4x+1$

② $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2}$

③ $\lim\limits_{x\to 4} \dfrac{x^2-2x-8}{x-4}$

④ $\lim\limits_{x\to -10} \dfrac{\sqrt{x+19}-3}{x+10}$

⑤ $\lim\limits_{x\to -1} \dfrac{x^2+2x-15}{x^2-2x-3}$

$$\frac{(x+5)\cancel{(x-3)}}{(x+1)\cancel{(x-3)}} = \frac{x+5}{x+1}$$

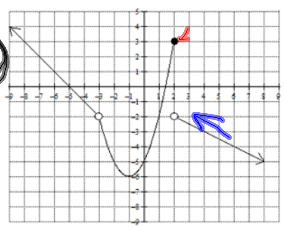
DNE

Aug 1-8:43 AM

Write the equation of the piece-wise defined function pictured to the right.

$mx+b$

	Equation of Each Piece		Constraint of Each Piece
①	$-x-5$	,	$x<-3$
$f(x)=$ ②	$(x+1)^2-6$	,	$-3<x\le 2$
③	$-\frac{1}{2}x-1$	,	$x>2$



Down 6
Left 1

$\lim x\to 2^+$

$$-\tfrac{1}{2}(2)-1=-2$$

$\lim x\to 2^-$

$$(2+1)^2-6$$

$$=3$$

Aug 3-8:20 AM

Use the equation that you just wrote to find each of the following limits. Confirm your results based on the graph. If a limit does not exist, state why.

a) $\lim_{x\to 2^+} f(x)$

$= -2$

b) $\lim_{x\to 2} f(x) = \text{DNE}$

b/c $\lim_{x\to 2^-} \neq \lim_{x\to 2^+}$

c) $\lim_{x\to -3} f(x)$

$= -2$

d) $\lim_{x\to -3^+} f(x)$

$= -2$

e) $\lim_{x\to -7} f(x)$

$= 2$

f) $\lim_{x\to -1} f(x)$

$= -6$

$-(-3)-5$

$(-3+1)^2-6$

$-(-7)-5$

$(-1+1)^2-6$

Aug 3-8:21 AM

Consider the function below to find each limit. If a limit does not exist, state why.

$$G(x) = \begin{cases} 2x^2+3x, & x \leq -2 \\ -\frac{1}{2}x+1, & x \geq -2 \end{cases}$$

a) $\lim_{x\to -2^-} G(x)$

$2(-2)^2+3(-2)$

$8-6$

$=2$

b) $\lim_{x\to -2^+} G(x)$

$-\frac{1}{2}(-2)+1$

$1+1$

$=2$

c) $\lim_{x\to -2} G(x)$

$=2$

Aug 3-8:20 AM

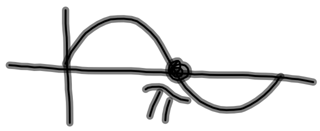
a. $\lim\limits_{x \to e} \frac{\ln x}{2x}$ $\frac{\ln e}{2e} = \boxed{\frac{1}{2e}}$

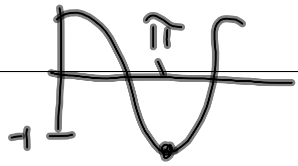
b. $\lim\limits_{x \to 5^-} \left(\frac{2}{5}x^2 + 2x\right)$ $= \frac{2}{5}(5)^2 + 2(5) = \boxed{20}$

c. $\lim\limits_{\theta \to \pi} \left(\sin^2\theta + 2\cos\theta\right)$ $= -2$

$[\sin(\pi)]^2 + 2\cos(\pi)$

$0^2 + 2(-1)$





Aug 3-8:43 AM

d. $\lim\limits_{\alpha \to \frac{5\pi}{3}} \frac{\tan\alpha}{\alpha^2}$ $\frac{\tan\left(\frac{5\pi}{3}\right)}{\left(\frac{5\pi}{3}\right)^2} = \frac{-\sqrt{3}}{\frac{25\pi^2}{9}} = -\sqrt{3} \cdot \frac{9}{25\pi^2}$

$\boxed{\frac{-9\sqrt{3}}{25\pi^2}}$

e. $\lim\limits_{x \to -2} \frac{x^2 - x - 6}{2x + 4}$ $= \frac{(x-3)\cancel{(x+2)}}{2\cancel{(x+2)}}$

$= \frac{x-3}{2} = \frac{-2-3}{2} = \frac{-5}{2}$

Aug 3-8:21 AM

Ms. Nina would like all of you to try logging on to PowerSchool. She wants you to check to make sure all of your classes are listed correctly.

You may also check your Calculus grade since that is up to date!

Aug 4-7:47 AM

f. $\lim_{x\to 3}\frac{x+5}{x^2-9} = \frac{x+5}{(x+3)(x-3)} = DNE$

$a=2x \quad b=3$

g. $\lim_{x\to \frac{3}{2}}\frac{8x^3-27}{2x-3} = \frac{\cancel{(2x-3)}(4x^2+6x+9)}{\cancel{2x-3}}$

$= 4x^2+6x+9 = 4(3/2)^2+6(3/2)+9$

$= \boxed{27}$

Aug 4-7:49 AM

SAME OPP Always +

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

Aug 4-8:05 AM

h. $\lim_{x\to -2}\frac{(\sqrt{2x+5}-1)}{x+2}\cdot\frac{(\sqrt{2x+5}+1)}{\sqrt{2x+5}+1}$

F: $2x+5$ O: $\cancel{\sqrt{2x+5}}$ I: $-\cancel{\sqrt{2x+5}}$ L: -1

$$=\lim_{x\to -2}\frac{2x+4}{(x+2)(\sqrt{2x+5}+1)}$$

$$=\lim_{x\to -2}\frac{2\cancel{(x+2)}}{\cancel{(x+2)}(\sqrt{2x+5}+1)}$$

$$=\lim_{x\to -2}\frac{2}{\sqrt{2x+5}+1}=\frac{2}{\sqrt{2(-2)+5}+1}=\boxed{1}$$

Aug 4-7:49 AM

i. $\lim\limits_{x\to 1} \frac{1-\sqrt{2x^2-1}}{x-1} \cdot \frac{(1+\sqrt{2x^2-1})}{(1+\sqrt{2x^2-1})}$

F O I L

$1 \quad \cancel{\sqrt{2x^2-1}} \quad \cancel{-\sqrt{2x^2-1}} \quad -(2x^2-1)$

$-2x^2+1$

$\lim\limits_{x\to 1} \frac{-2x^2+2}{(x-1)(1+\sqrt{2x^2-1})} = \frac{-2(x^2-1)}{(x-1)(1+\sqrt{2x^2-1})}$

$\lim\limits_{x\to 1} \frac{-2(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(1+\sqrt{2x^2-1})} = \lim\limits_{x\to 1} \frac{-2(x+1)}{(1+\sqrt{2x^2-1})}$

$= \frac{-2(1+1)}{(1+\sqrt{2(1)^2-1})} = \frac{-4}{2} = \boxed{-2}$

Aug 4-7:49 AM

j. $\lim\limits_{x\to 0} \frac{\frac{1}{x+2}+\frac{1}{x}}{x}$

$\frac{x}{x} \cdot \frac{1}{x+2} + \frac{1}{x} \cdot \frac{(x+2)}{(x+2)}$

$\frac{x}{x(x+2)} + \frac{x+2}{x(x+2)}$

$\lim\limits_{x\to 0} \frac{\frac{2x+2}{x(x+2)}}{\frac{x}{1}} = \lim\limits_{x\to 0} \frac{2x+2}{x(x+2)} \cdot \frac{1}{x}$

$= \lim\limits_{x\to 0} \frac{2x+2}{x^2(x+2)} = \lim\limits_{x\to 0} \frac{2(x+1)}{x^2(x+2)} \qquad \frac{2}{0}$

DNE

Aug 4-7:49 AM

k. $\lim_{x\to 2^+} \frac{3x^2+7x+2}{x^2-4} = \frac{(3x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$

$\lim_{x\to 2^+} \frac{3x+1}{x-2}$ $\quad \frac{3(2)+1}{2-2} = \frac{7}{0}$ VA.

$\frac{3(2.1)+1}{2.1-2} = \frac{+}{+} =$ Pos

$\lim_{x\to 2^+} \frac{3x+1}{x-2} = \infty$

Aug 4-7:49 AM

l. $\lim_{x\to 3^+} \frac{2x+5}{x-3}$ $\quad \frac{2(3)+5}{3-3} = \frac{11}{0}$

VA.

m. $\lim_{x\to 3^-} \frac{2x+5}{x-3}$

$\frac{2(3.1)+5}{3.1-3}$ $\qquad \frac{2(2.9)+5}{2.9-3}$

$\lim_{x\to 3^+} \frac{2x+5}{x-3} = \infty$ $\qquad \lim_{x\to 3^-} \frac{2x+5}{x-3} = -\infty$

Aug 4-7:50 AM

If $f(x) = 2x^2 - 3x + 4$, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Aug 4-7:50 AM

Due Monday - HW #3

#1-11, 14-20

Aug 4-8:49 AM