Based on what you have just observed, what inference can you make about the value of the limit $\lim_{\theta \to 0} \frac{\sin c\theta}{c\theta}$, where c is any constant?

$$\lim_{\theta \to 0} \frac{\sin c\theta}{c\theta} = 1$$

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These two special trigonometric functions derived above can often be used to find limits of trigonometric functions that cannot be evaluated by direct substitution nor by rewriting the function using identities.

$$\lim_{\theta \to 0} \frac{\sin c\theta}{c\theta} = 1 \qquad \lim_{\theta \to 0} \frac{1-\cos c\theta}{c\theta} = 0$$

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1. $\lim_{x\to 0} \frac{e^x \cos x}{4}$

$\frac{e^0 \cos 0}{4} = \frac{1 \cdot 1}{4}$

$= \frac{1}{4}$

2. $\lim_{\theta\to 0} \frac{\sin 4\theta}{4\theta}$ $\frac{4}{4} = 1$

$4 \cdot \lim_{\theta\to 0} \frac{\sin 4\theta}{4\theta}$

$4 \cdot 1 = 4$

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3. $\lim_{x\to 0} \frac{\sin 2x}{3x} = \frac{1}{3} \lim_{x\to 0} \frac{2\sin 2x}{2x}$

$\frac{2}{3} \lim_{x\to 0} \frac{\sin 2x}{2x}$

$\frac{2}{3} \cdot 1$

$\frac{2}{3}$

4. $\lim_{\theta\to 0} \frac{2\sin 5\theta}{3\theta}$

$\frac{2}{3} \lim_{\theta\to 0} \frac{5}{1} \frac{\sin 5\theta}{5\theta}$

$\frac{10}{3} \lim_{\theta\to 0} \frac{\sin 5\theta}{5\theta}$

$\frac{10}{3}$

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5. $\lim_{\theta\to 0}\frac{\tan\theta}{\theta}$ $= \lim_{\theta\to 0}\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\theta}{1}} = \lim_{\theta\to 0}\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\theta}$

$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta\cos\theta}$

$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\frac{1}{\cos\theta}$

$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\lim_{\theta\to 0}\frac{1}{\cos\theta} = 1$

$1 \cdot \frac{1}{1}$

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6. $\lim_{\theta\to 0}\frac{2-2\cos^2\theta}{\theta}$

$= \lim_{\theta\to 0}\frac{2(1-\cos^2\theta)}{\theta}$

$= \lim_{\theta\to 0}\frac{2\sin^2\theta}{\theta}$

$= \lim_{\theta\to 0}\frac{2\sin\theta\sin\theta}{\theta}$

$= \lim_{\theta\to 0} 2\sin\theta\cdot\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$

$0 \cdot 1 = 0$

7. $\lim_{\theta\to 0}\frac{1-\cos\theta+\sin 2\theta}{\theta}$

$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} + 2\lim_{\theta\to 0}\frac{\sin 2\theta}{2\theta}$

$0 + 2$

2

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8. $\lim_{\theta\to 0}\dfrac{\theta\csc\theta+1}{\theta\csc\theta}$

$\lim_{\theta\to 0}\dfrac{\theta\csc\theta}{\theta\csc\theta}+\lim_{\theta\to 0}\dfrac{1}{\theta\csc\theta}$

$\lim_{\theta\to 0}1+\lim_{\theta\to 0}\dfrac{\sin\theta}{\theta}$

$1+1$

2

9. $\lim_{x\to 0}\dfrac{\sin x-\sin x\cos x}{x^2}$

$\lim_{x\to 0}\dfrac{\sin x(1-\cos x)}{x^2}$

$\lim_{x\to 0}\dfrac{\sin x}{x}\cdot\lim_{x\to 0}\dfrac{1-\cos x}{x}$

$1\cdot 0$

0

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$\lim_{x\to 0}\dfrac{\tan 4x}{x}=\lim_{x\to 0}\dfrac{\frac{\sin 4x}{\cos 4x}}{\frac{x}{1}}\cdot\dfrac{1}{x}$

$\lim_{x\to 0}\dfrac{\sin 4x}{x\cos 4x}$

$4\cdot\lim_{x\to 0}\dfrac{\sin 4x}{4x}\cdot\lim_{x\to 0}\dfrac{1}{\cos 4x}$

$4\cdot 1$

$=4$

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$$\lim_{x\to 0} \frac{\sin^2 x - \sin x \cos x}{x\cos x}$$

$$\lim_{x\to 0} \frac{\sin x}{x}\cdot\frac{\sin x}{\cos x} - \lim_{x\to 0} \frac{\sin x \cancel{\cos x}}{x \cancel{\cos x}}$$

$$\left[\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{\cos x}\right] - 1$$

$\frac{\sin 0}{\cos 0}$

$1 \cdot 0 = 0$

$= -1$

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$$\lim_{x\to 0} \frac{\sin 2x - 3x^2 + 2x}{6x}$$

#52

$$\lim_{x\to 0}\frac{\sin 2x}{6x} - \lim_{x\to 0}\frac{3x^{\cancel{2}}}{6\cancel{x}} + \lim_{x\to 0}\frac{2\cancel{x}}{6\cancel{x}}$$

$$\frac{\cancel{2}}{6}\cdot\underbrace{\frac{\sin 2x}{2x}}_{1} - 0 + \frac{1}{3}$$

$\frac{3(0)}{6}$

$\frac{1}{3} - 0 + \frac{1}{3} = \boxed{\frac{2}{3}}$

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#55 $\lim_{x\to 0} \frac{5x - \sin 10x + 15x^2}{2x}$

$\lim_{x\to 0} \frac{5x}{2x} - \lim_{x\to 0} \frac{\sin 10x}{2x} + \lim_{x\to 0} \frac{15x^2}{2x}$

$\frac{5}{2} - \frac{1}{2}\cdot 10\cdot \frac{\sin 10x}{10x} \qquad \frac{15x}{2}$

$5 \qquad 1 \qquad 0$

$\frac{5}{2} - 5 + 0$

$= -2.5$ or $-\frac{5}{2}$

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#48 $\lim_{x\to 0} \frac{7x}{\sin 3x}$

$\lim_{x\to 0} \frac{7}{1\cdot 3} \cdot \frac{3\cdot x}{\sin 3x}$

$\frac{7}{3} \cdot 1$

$\frac{7}{3}$

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#46 $\lim_{x\to 0} \frac{\sin 3x}{12x\cos x}$

$\lim_{x\to 0} \frac{\sin 3x}{12x} \cdot \lim_{x\to 0} \frac{1}{\cos x}$

$\frac{1\cdot 3}{12} \frac{\sin 3x}{3\cdot x} \qquad \frac{1}{\cos 0} \quad \frac{1}{1}$

$\cdot \quad 1$

$\boxed{\frac{1}{4}}$

Aug 18-8:10 AM

#54 $\lim_{x\to 0} \frac{5x^3 - 2x^2}{\sin x}$

$\lim_{x\to 0} \frac{x\cdot(5x^2 - 2x)}{\sin x}$

$\underbrace{\frac{x}{\sin x}}_{1} \cdot \underbrace{(5x^2 - 2x)}_{0}$

$\cdot$

$\boxed{0}$

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