

$$\lim_{x \rightarrow -1} \frac{(x+3)^3 - 8}{x+1}$$

$a = (x+3)$
 $b = 2$

$(a - b)(a^2 + ab + b^2)$

$$\lim_{x \rightarrow -1} \frac{\cancel{(x+3-2)} \left((x+3)^2 + 2(x+3) + 4 \right)}{\cancel{x+1}}$$

$(-1+3)^2 + 2(-1+3) + 4$
 $4 + 4 + 4$
 $\boxed{12}$

Aug 21-7:52 AM

4. Consider the following piece-wise defined function: $f(x) = \begin{cases} -3 + 2x - x^2 & \text{if } x < -1 \\ -3 + 2x - x^2 & \text{if } x \geq -1 \end{cases}$

Find the following limits:

a) $\lim_{x \rightarrow -1^-} f(x) = -6$ b) $\lim_{x \rightarrow -1^+} f(x) = -6$ c) $\lim_{x \rightarrow -1} f(x) = -6$ d) $\lim_{x \rightarrow 2} f(x) = -11$ e) $\lim_{x \rightarrow 0} f(x) = -3$

Find $\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3} = \frac{(\sqrt{x}+3)(\sqrt{x}-3)}{(\sqrt{x}-3)(\sqrt{x}+3)} = \frac{x-9}{x-9} = 1$ $\lim_{x \rightarrow 1} \frac{4x^4 - 5x^2 + 1}{x^2 + 2x - 3} = \frac{(x+1)(x-1)(2x+1)(2x-1)}{(x^2-1)(4x^2-1)} = \frac{2x+1}{4x-1} = \frac{3}{2}$

$\lim_{x \rightarrow -2} \frac{x}{x+4} + 1 = \frac{-2}{-2+4} + 1 = \frac{-2}{2} + 1 = -1 + 1 = 0$ $\lim_{x \rightarrow 2} \frac{2}{x^2} - \frac{1}{2} = \frac{2}{4} - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$

$\lim_{x \rightarrow -2} \frac{2x+4}{x+4} \cdot \frac{1}{x+2} = \frac{2(x+2)}{x+4} \cdot \frac{1}{x+2} = \frac{2}{x+4} = \frac{2}{-2+4} = 1$ $\lim_{x \rightarrow 2} \frac{4-x^2}{2x^2} \cdot \frac{1}{x-2} = \frac{-(x^2-4)}{2x^2} \cdot \frac{1}{x-2} = \frac{-(x+2)(x-2)}{2x^2} \cdot \frac{1}{x-2} = \frac{-(x+2)}{2x^2} = \frac{-4}{8} = -\frac{1}{2}$

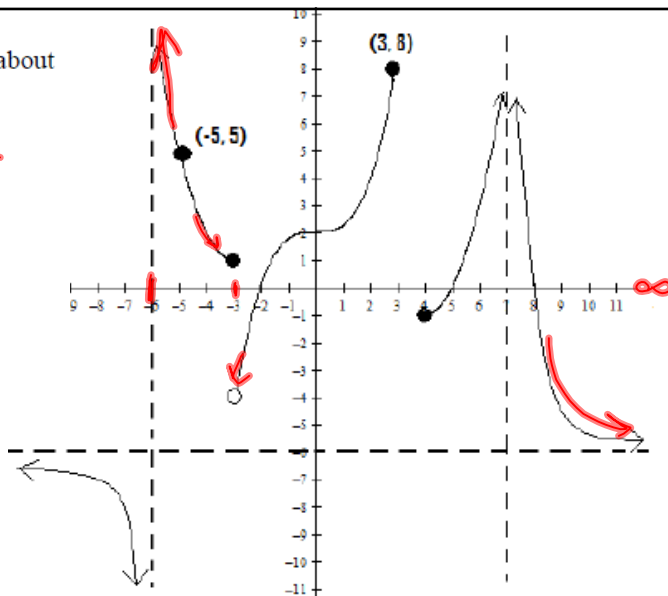
Aug 21-8:10 AM

1. Which of the following statements is/are true about the graph of $H(x)$?

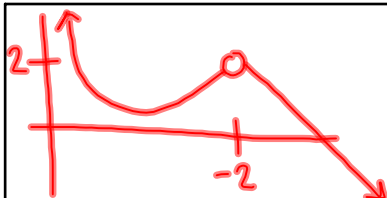
I. $\lim_{x \rightarrow -3^-} H(x) = H(-3)$ **TRUE**

II. $\lim_{x \rightarrow \infty} H(x) = -6$ **TRUE**
x → *y-values*

III. $\lim_{x \rightarrow -6^+} H(x) = -\infty$
FALSE



Aug 21-8:13 AM



$$G(x) = \begin{cases} 2x^2 + 3x, & x < -2 \\ -\frac{1}{2}x + 1, & x > -2 \end{cases}$$

a) $\lim_{x \rightarrow -2^-} G(x)$

2

b) $\lim_{x \rightarrow -2^+} G(x)$

2

c) $\lim_{x \rightarrow -2} G(x)$

2

$G(-2) = \text{undefined}$

↑
Closed Circle

Aug 21-8:13 AM

Find $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin x}{3x}$.

$$\lim_{x \rightarrow 0} \frac{\sin x}{3x}$$

$$\frac{\sin(\frac{\pi}{6})}{3(\frac{\pi}{6})} = \frac{1/2}{\pi/2}$$

$$\frac{1}{2} \cdot \frac{2}{\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$$

$$\frac{1}{3} \underbrace{\frac{\sin x}{x}}_1$$

$$\boxed{\frac{1}{3}}$$

Aug 22-7:41 AM

$$\lim_{x \rightarrow 2} f(x) \text{ if } f(x) = \begin{cases} 2x^2 - 4x, & x < 2 \\ 4 \sin\left(\frac{\pi x}{4}\right), & x > 2 \end{cases}$$

DNE

$f(2) \rightarrow \text{undefined}$

Aug 22-7:43 AM

$$\text{Find } \lim_{x \rightarrow 0} \frac{4x + \sin x}{2x}$$

$$\frac{4x}{2x} + \frac{\sin x}{2x}$$

$$2 + \frac{1}{2} \frac{\sin x}{x}$$

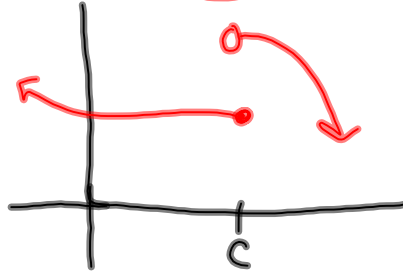
$$2 + \frac{1}{2}$$
$$\boxed{2.5}$$

Aug 22-7:45 AM

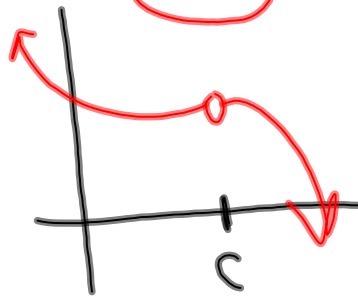
$$\lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7}$$

Aug 21-8:14 AM

Draw the graph of a function that is defined at $x=c$ but the limit as x approaches c does not exist.



Draw the graph of a function that is not defined at $x=c$ but the limit as x approaches c does exist.



Aug 21-8:14 AM

Draw a graph that meets the following criteria.

$$\lim_{x \rightarrow 3^-} f(x) = \infty$$

$$\lim_{x \rightarrow 3^+} f(x) = -\infty$$

$$f(-2) = 4$$

$$\lim_{x \rightarrow -2} f(x) = 1$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

Aug 22-7:51 AM

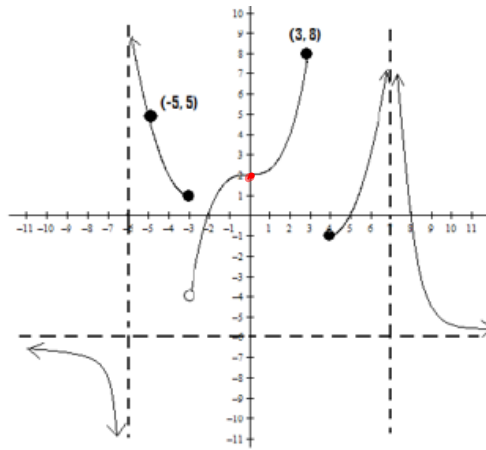
1. Which of the following statements is/are true about the graph of $H(x)$?

~~I. $\lim_{x \rightarrow -3^+} H(x) = H(-3)$~~

II. $\lim_{x \rightarrow \infty} H(x) = -6$ ✓

III. $\lim_{x \rightarrow -6^-} H(x) = -\infty$ ✓

- A. I and II only
- B. II only
- C. I and III only
- D. II and III only**
- E. I, II, and III



2. Which of the following limit(s) do(es) not exist?

I. $\lim_{x \rightarrow 7} H(x)$ **DNE**

II. $\lim_{x \rightarrow 3} H(x)$ **DNE**

III. $\lim_{x \rightarrow 0} H(x)$

- A. I only
- B. I and II only**
- C. II only
- D. II and III only
- E. III only

Aug 25-7:55 AM

3. If $g(x) = \begin{cases} e^x(x+1), & x < -2 \\ \cos(\pi x), & x > -2 \end{cases}$, which of the following statements is/are true?

I. $g(-2)$ is undefined. **TRUE**

II. $\lim_{x \rightarrow -2^-} g(x) = -\frac{1}{e^2}$ **TRUE**

III. $\lim_{x \rightarrow -2} g(x)$ exists. **FALSE**

- A. I and II only**
- B. II only
- C. II and III only
- D. I and III only
- E. I, II, and III

$e^{-2}(-2+1)$

$-e^{-2}$

$-\frac{1}{e^2}$

$\cos(-2\pi)$

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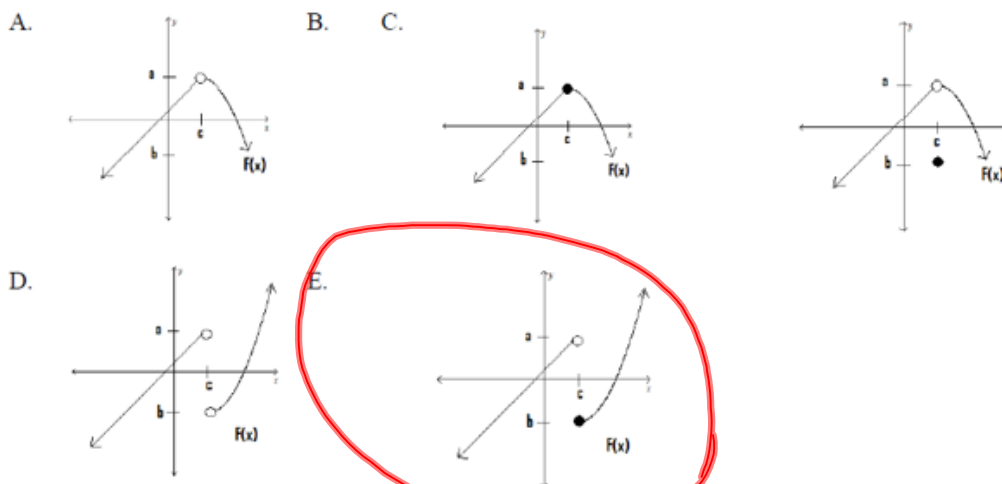
4. Find $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3}$. $\frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} = \frac{\cancel{x+1}-4}{(\cancel{x-3})(\sqrt{x+1} + 2)} = \frac{1}{\sqrt{x+1} + 2}$

- A. 4 B. $\frac{1}{2}$ C. $\frac{1}{4}$ D. -4 E. Limit does not exist

$$\frac{1}{\sqrt{3+1} + 2} = \frac{1}{4}$$

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5. Which one of the following graphs shows that $F(c)$ is defined but the $\lim_{x \rightarrow c} F(x)$ does not exist?



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10. $\lim_{x \rightarrow 3} -\sqrt{x+3}$

$$-\sqrt{3+3}$$

$$\boxed{-\sqrt{6}}$$

11. $\lim_{x \rightarrow \frac{3}{2}} \frac{-x-3}{x^2+x+1}$

$$\frac{-\frac{3}{2} - 3}{\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 1} = \frac{-\frac{9}{2}}{\frac{19}{4}}$$

$$\frac{9}{4} + \frac{6}{4} + \frac{4}{4}$$

$$\boxed{-\frac{18}{19}}$$

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12. $\lim_{x \rightarrow 0} \frac{4 \cdot \frac{1}{-4+x} + \frac{1}{4} \cdot (-4+x)}{x}$

$$\frac{4 + -4 + x}{4(-4+x)}$$

$$\frac{x}{-4(-4+x)}$$

$$= \frac{1}{4(-4+x)} = \boxed{-\frac{1}{16}}$$

13. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$

$$\frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$$

$$-(x+1)$$

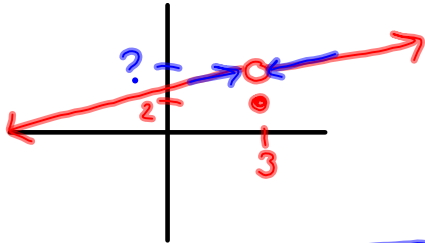
$$-(2+1)$$

$$\boxed{-3}$$

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14. $f(3) = 2$

$$\lim_{x \rightarrow 3} f(x), f(x) = \begin{cases} 2 + \frac{x}{2}, & x \neq 3 \\ 2, & x = 3 \end{cases}$$



$$2 + \frac{3}{2} = \boxed{3.5}$$

15.

$$\lim_{x \rightarrow -2} f(x), f(x) = \begin{cases} x^2, & x \leq -2 \\ -\frac{x}{2} + 3, & x > -2 \end{cases}$$

$$\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^+} f(x)$$

$$\begin{aligned} (-2)^2 &= -\frac{-2}{2} + 3 \\ 4 &= \frac{4}{4} \\ &\boxed{4} \end{aligned}$$

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16. $\lim_{x \rightarrow 3^-} \frac{4x}{x-3}$

$$\frac{-4(2.9)}{2.9-3} = \frac{\text{Neg}}{\text{Neg}} = +$$

$$\infty$$

17.

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+6} - 3}{x-3}$$

Aug 25-7:57 AM

6. Draw a graph of a function, $H(x)$, that meets the following criteria.

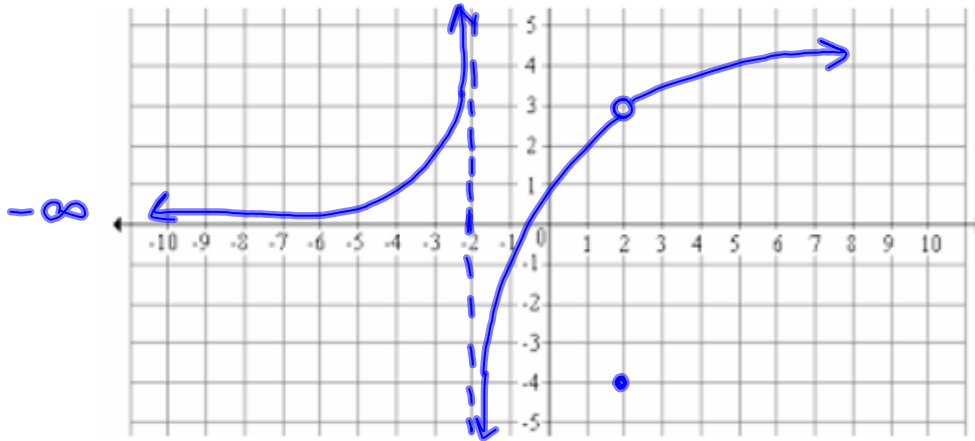
✓ $\lim_{x \rightarrow 2^-} H(x) = \infty$

✓ $\lim_{x \rightarrow 2^+} H(x) = -\infty$

$\lim_{x \rightarrow 2} H(x) = 3$

✓ $\lim_{x \rightarrow -\infty} H(x) = 0$

$H(2) = -4$



Aug 25-8:39 AM

Given $\lim_{x \rightarrow 3} F(x) = 4$ and $\lim_{x \rightarrow 3} G(x) = -2$, find the following limits:

7. $\lim_{x \rightarrow 3} 3F(x) + 2G(x)$

8

8. $\lim_{x \rightarrow 3} G(x) + \sqrt{F(x)}$

0

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9. Consider the functions below to answer the following questions.

$$F(x) = \begin{cases} x^2 + 2|x|, & x < -2 \\ 3x + a, & x > -2 \end{cases}$$

$$G(x) = \frac{2x^2 - 5x - 3}{x^2 - 9}$$

- a. Find the value of $\lim_{x \rightarrow -2^-} F(x)$. Show your work.

$$(-2)^2 + 2|-2| = 8$$

- b. Is $G(3) = \lim_{x \rightarrow 3} G(x)$? Show your work and explain your reasoning.

↑
undef. ↑
#

- c. In order for $\lim_{x \rightarrow -2} F(x)$ to exist, what two limits must be equal? Find the value(s) of a for which this limit exists. Show your work.

$$\begin{aligned} 3(-2) + a &= 8 \\ -6 + a &= 8 \\ a &= 14 \end{aligned}$$

Aug 25-8:46 AM

20. Give an example of a two-sided limit of a piece-wise function where the limit does not exist.

21. Give an example of a limit that evaluates to 7.

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Extra Credit: (5 points, MUST SHOW WORK!) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ if $f(x) = 3x^2 + 4x - 5$

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