If $f(x) = 2x^2 - 3x + 4$, find $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$. Difference Quotient

$f(x+h) = 2(x+h)^2 - 3(x+h) + 4$

$(x+h)(x+h)$

$= 2(x^2 + 2xh + h^2) - 3(x+h) + 4$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 4$

$\lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 4 - (2x^2 - 3x + 4)}{h}$

$\lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h + \cancel{4} - \cancel{2x^2} + \cancel{3x} - \cancel{4}}{h}$

$\lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h\to 0} 4x + \cancel{2h}^{0} - 3$

$= \boxed{4x - 3}$

Aug 7-7:45 AM

If $f(x) = 3x^2 + 4x - 7$ find $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = 3(x+h)^2 + 4(x+h) - 7$

$= 3(x^2 + 2xh + h^2) + 4x + 4h - 7$

$= 3x^2 + 6xh + 3h^2 + 4x + 4h - 7$

$\lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{4x} + 4h - \cancel{7} - \cancel{3x^2} - \cancel{4x} + \cancel{7}}{h}$

$\lim_{h\to 0} 6x + 3h + 4$

$= \boxed{6x + 4}$

Aug 7-8:14 AM

Properties of Limits

Suppose $\boxed{\lim_{x \to a} f(x) = L}$ and $\boxed{\lim_{x \to a} g(x) = M}$. Find each of the following limits in terms of L and M.

1. $\lim_{x \to a} [f(x) + g(x)]$	$= \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$ $\boxed{L + M}$
2. $\lim_{x \to a} [f(x) - g(x)]$	$= \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$ $\boxed{L - M}$

Aug 7-7:46 AM

Properties of Limits

Suppose $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. Find each of the following limits in terms of L and M.

3. $\lim_{x \to a} \frac{f(x)}{g(x)}$	$\frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \boxed{\frac{L}{M}}$
4. $\lim_{x \to a} [f(x) \cdot g(x)]$	$\lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$ $\boxed{L \cdot M}$

Aug 7-7:46 AM

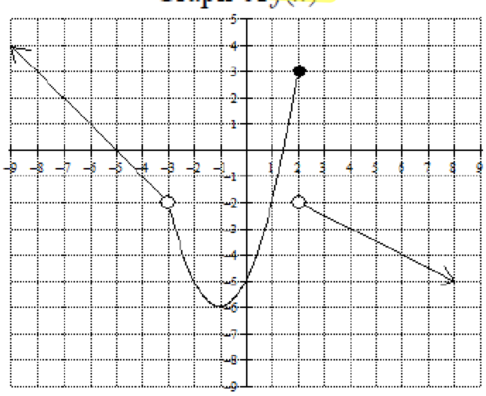
Properties of Limits

Suppose $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. Find each of the following limits in terms of L and M.

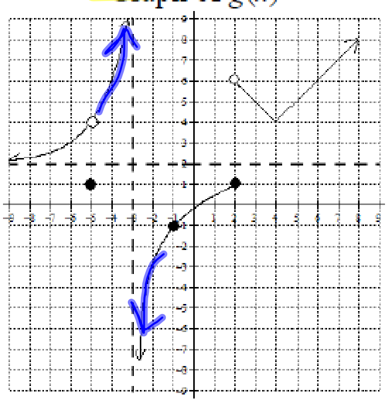
5. $\lim_{x \to a} [c \cdot f(x)]$	$= c \cdot \lim_{x \to a} f(x)$ $= \boxed{c \cdot L}$
6. $\lim_{x \to a} [f(x)]^p$	$\left[\lim_{x \to a} f(x)\right]^p = L^p$
7. $\lim_{x \to a} c$	$= c$

Aug 7-7:47 AM

Graph of $f(x)$



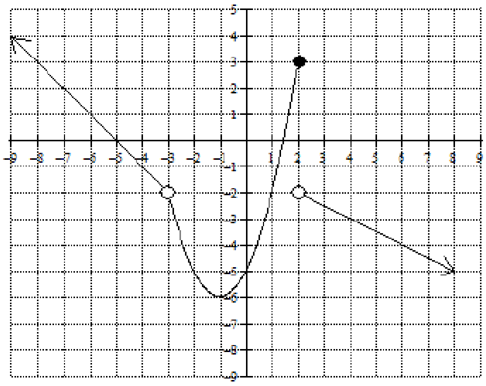
Graph of $g(x)$



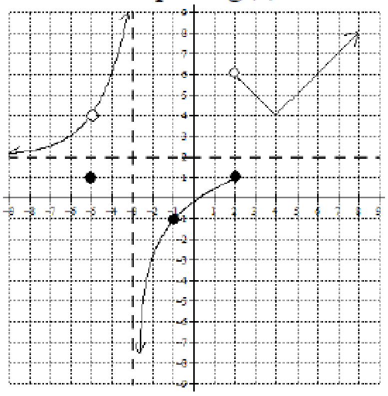
$\lim_{x \to 2^-} [f(x) + g(x)]$	$\lim_{x \to -1} [2f(x) - 3g(x)]$	$\lim_{x \to -3} [f(x) - g(x)]$
$\lim_{x \to 2^-} f(x) + \lim_{x \to 2^-} g(x)$	$2(-6) - 3(-1)$	$-2 -$ DNE
$3 + 1$	$\boxed{-9}$	DNE
$\boxed{4}$		

Aug 7-7:48 AM

Graph of $f(x)$



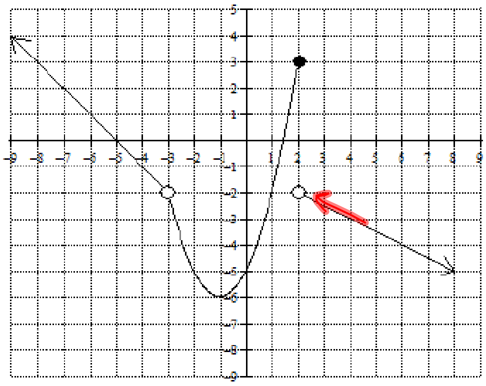
Graph of $g(x)$



$\lim\limits_{x\to 6} \dfrac{-2f(x)}{g(x)}$	$\lim\limits_{x\to 4} 2[f(x)g(x)]$	$\lim\limits_{x\to -2} [f(x)]^2$
$\dfrac{-2(-4)}{6} = \dfrac{8}{6} = \dfrac{4}{3}$	$2[-3\cdot 4]$ $\boxed{-24}$	$[-5]^2 = \boxed{25}$

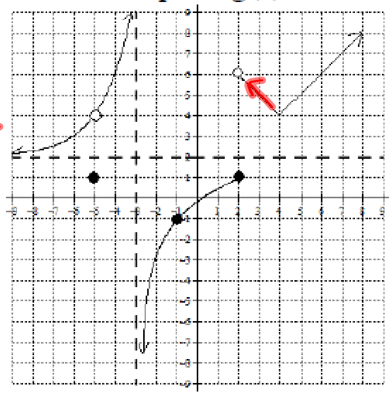
Aug 7-7:47 AM

Graph of $f(x)$



Graph of $g(x)$

Left



Right

$\lim\limits_{x\to 2^+} \sqrt{2g(x)}$	$\lim\limits_{x\to 2^+} \dfrac{f(x)}{g(x)}$	$\lim\limits_{x\to -1} [f(x)-g(x)]$
$\sqrt{2\cdot 6}$ $\sqrt{12} = \boxed{2\sqrt{3}}$	$\dfrac{-2}{6} = \boxed{\dfrac{-1}{3}}$	$-6 - -1$ $\boxed{-5}$

Aug 7-7:48 AM

10. $\lim_{x\to 0} \dfrac{\frac{2}{2}\cdot\frac{1}{x+2}-\frac{1}{2}\cdot\frac{x+2}{x+2}}{x}$

$$\frac{\frac{2}{2(x+2)}-\frac{x+2}{2(x+2)}}{x}$$

$$\frac{\frac{-x}{2(x+2)}}{\frac{x}{1}} = \frac{-\cancel{x}}{2(x+2)}\cdot\frac{1}{\cancel{x}} = \frac{-1}{2(x+2)}$$

$$\boxed{\frac{-1}{4}}$$

Aug 7-8:43 AM

9. $\lim_{x\to 0} \dfrac{5x^3+8x^2}{3x^4-16x^2}$

$$\frac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)}$$

$$\frac{5x+8}{3x^2-16} \quad \text{Plug In}$$

Aug 7-8:50 AM

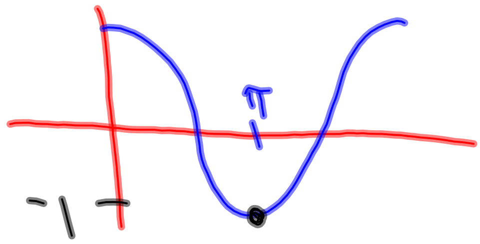
15. $\lim_{x \to 3} e^x \cos\left(\frac{\pi x}{3}\right)$

$e^3 \cos\left(\frac{3\pi}{3}\right)$

$e^3 \cos(\pi)$

$e^3(-1)$

$\boxed{-e^3}$



Aug 7-7:49 AM

5. $\lim_{\theta \to \frac{\pi}{6}} \theta^2 \tan\theta$

$\left(\frac{\pi}{6}\right)^2 \tan\left(\frac{\pi}{6}\right)$

$\left(\frac{\pi}{6}\right)^2 \frac{1}{\sqrt{3}}$

$\frac{\pi^2}{36\sqrt{3}}$

Aug 8-8:09 AM

Angle θ				
Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
180	π	0	-1	0
270	$\frac{3\pi}{2}$	-1	0	undefined
360	2π	0	1	0

Aug 8-8:15 AM

11. $\lim_{x\to 0} \frac{(2+x)^3-8}{x}$

$a^3-b^3=$

$(a-b)(a^2+ab+b^2)$

$a=(2+x)$

$b=2$

$$\lim_{x\to 0} \frac{(2+x-2)\left((2+x)^2+2(2+x)+4\right)}{x}$$

$$\lim_{x\to 0} \frac{\cancel{x}\left((2+x)^2+2(2+x)+4\right)}{\cancel{x}}$$

$(2+0)^2+2(2+0)+4 = \boxed{12}$

Aug 8-8:10 AM

14. $\lim_{x\to 2} f(x)$ if $f(x)=\begin{cases} 2x^2-4x, & x<2 \\ 4\sin\left(\frac{\pi x}{4}\right) & x>2 \end{cases}$

$\lim_{x\to 2^-} 2(2)^2-4(2) = 0$

$\lim_{x\to 2^+} 4\sin\left(\frac{\pi 2}{4}\right)$

$= 4\sin\left(\frac{\pi}{2}\right)$

$= 4(1) = 4$

$\lim_{x\to 2} f(x) = \text{DNE}$

Aug 8-8:28 AM

19. $\lim_{x\to \boxed{0}} \dfrac{\frac{2}{2}\cdot\frac{1}{x-2}+\frac{1}{2}\cdot\frac{x-2}{x-2}}{x}$

$$\frac{\frac{2}{2(x-2)}+\frac{x-2}{2(x-2)}}{x} = \frac{\frac{x}{2(x-2)}}{\frac{x}{1}}$$

$$\frac{\cancel{x}}{2(x-2)}\cdot\frac{1}{\cancel{x}} = \frac{1}{2(x-2)} = \boxed{-\frac{1}{4}}$$

Aug 8-8:32 AM

Limits Algebraically

DNE $\frac{1}{4}$ $\frac{6}{5}$ 2

2   7   3

undefined   2   7

2   2x   DNE

3

-6   3 $\frac{1}{6}$ 2

7   2

3   2   3

3   6   3   14

7

Worksheet 2

2   -4

-1 $\frac{4}{5}$

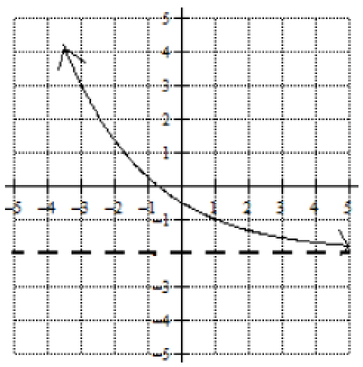
-4   -5

$\frac{1}{4}$ $\frac{1}{2\sqrt{3}}$ 12

$-\frac{1}{4}$

Aug 8-8:39 AM

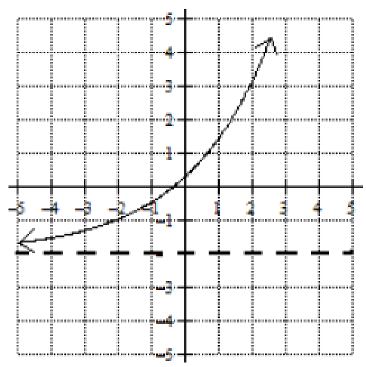
Decay $f(x) = \left(\frac{2}{3}\right)^{x-1} - 2$



HA.

$\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = -2$

Growth $f(x) = \left(\frac{3}{2}\right)^{x+2} - 2$

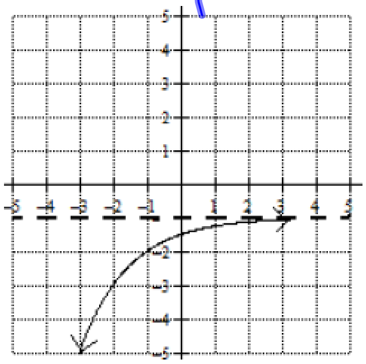


$\lim_{x\to-\infty} f(x) = -2$ $\lim_{x\to\infty} f(x) = \infty$

Aug 10-7:48 AM

Growth $f(x) = -\left(\frac{1}{2}\right)^{x+1} - 1$

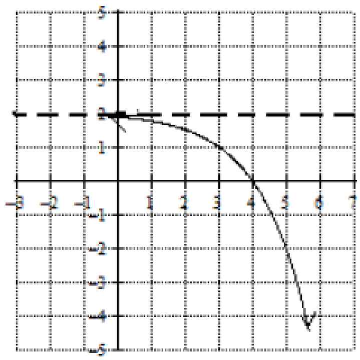
r



HA

$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = -1$

Decay $f(x) = -\left(\frac{1}{2}\right)^{-x+3} + 2$



$\lim_{x\to-\infty} f(x) = 2$ $\lim_{x\to\infty} f(x) = -\infty$

Aug 10-7:49 AM

In order to determine a limit as x approaches $-\infty$ or ∞ for an exponential function, you have to determine what the graph will look like. Based on what we have seen above, what are the three possible results of such a limit for an exponential function?

∞	$-\infty$	H.A.

Aug 10-7:49 AM

By studying the graphs above, remind yourself of the four rules determining if the function will be a growth or decay function.

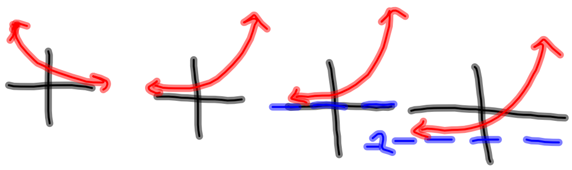
1. $b>1$ and 0 or 2 reflections → Growth
2. $b>1$ and 1 reflections → Decay
3. $0<b<1$ and 0 or 2 reflections → Decay
4. $0<b<1$ and 1 reflection → Growth

Aug 10-7:49 AM

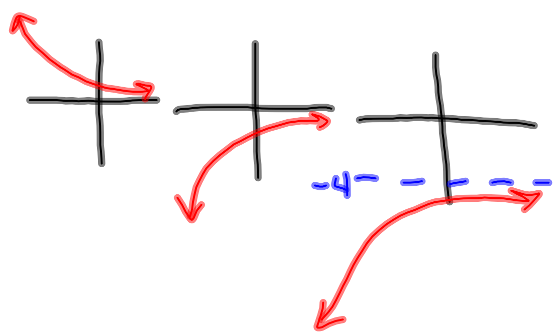
L/R U/D Down 4

Determine the limits of each of the following exponential functions.

1. $\lim_{x\to\infty}\left(\frac{2}{3}\right)^{-x-1}-2 = \infty$



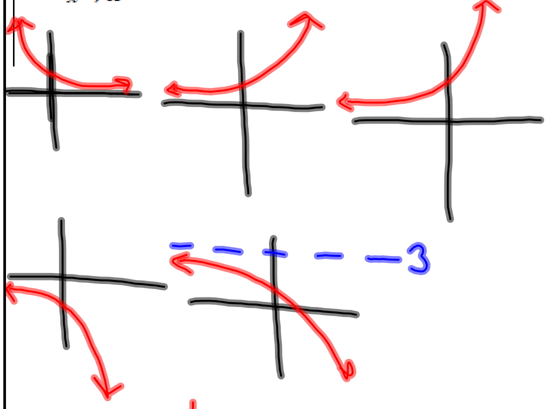
2. $\lim_{x\to-\infty} -(0.4)^{x}-4 = -\infty$



Aug 10-7:50 AM

u/p y/R left 2

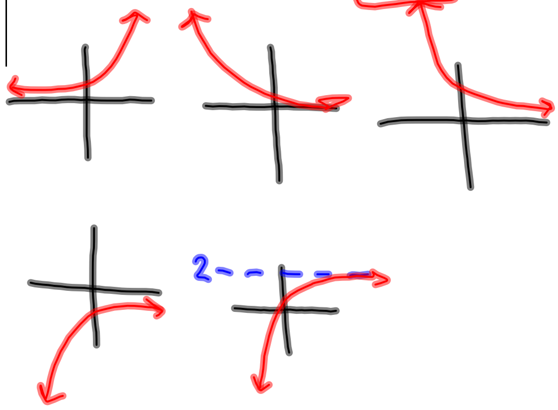
3. $\lim_{x\to\infty} -\left(\frac{2}{3}\right)^{-x+2} + 3$ ← up 3



$= \boxed{-\infty}$

u/p right 1

4. $\lim_{x\to\infty} -2^{-x-1} + 2 = \boxed{2}$



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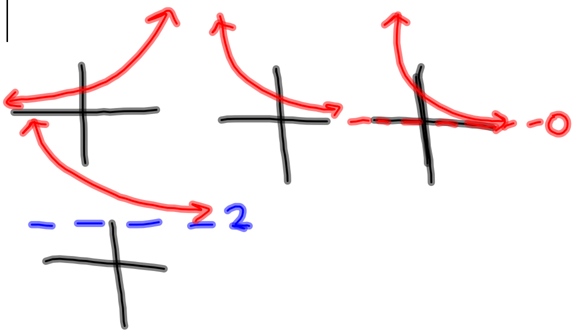
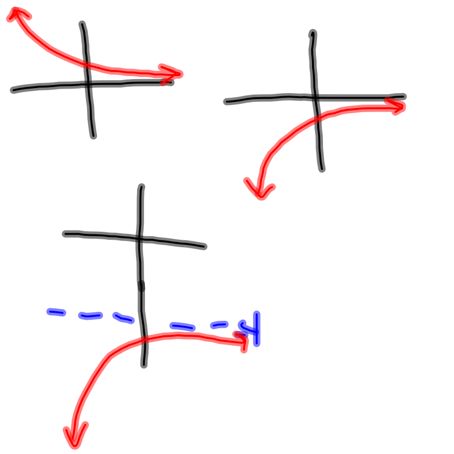
5. $\lim_{x\to-\infty} e^{-x-1} + 2 = \infty$



6. $\lim_{x\to\infty} -(0.4)^{x} - 4 = \boxed{-4}$

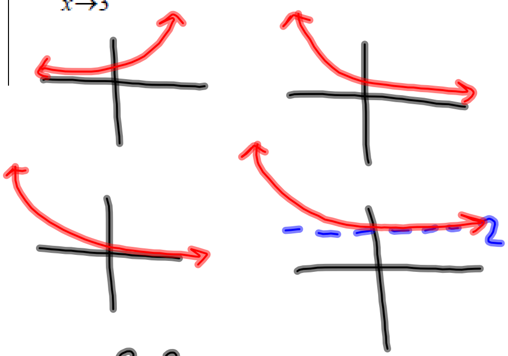


Aug 10-7:50 AM

$e^{-x+2} \quad +2$

7. $\lim_{x\to 3}\left(e^{2-x}+2\right)$

8. $\lim_{x\to -2}\left[\left(\frac{1}{2}\right)^{-x-3}+3\right]$



$e^{2-3}+2$

$e^{-1}+2=\boxed{\frac{1}{e}+2}$

$\frac{1}{2}^{-(-2)-3}+3$

$\frac{1}{2}^{-1}+3$

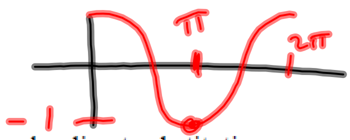
$\frac{1}{\frac{1}{2}}$

$1\cdot\frac{2}{1}=2+3=\boxed{5}$

Aug 10-7:50 AM

Limits of Trigonometric Functions

An Analytical Approach



We have already looked at how to evaluate limits of trigonometric functions by direct substitution, provided that the function is defined and continuous at θ. Find each of the limits below.

$\lim_{\theta\to\frac{2\pi}{3}}\frac{\sin 3\theta}{3\theta}=0$

$\frac{\sin \cancel{3}\left(\frac{2\pi}{\cancel{3}}\right)}{\cancel{3}\left(\frac{2\pi}{\cancel{3}}\right)}=\frac{0}{2\pi}$

$=0$

$\lim_{\theta\to\pi} 2\cos^2\theta=\boxed{2}$

$2(\cos\theta)^2$

$=2(\cos\pi)^2$

$=2(-1)^2$

$=2$

Aug 11-8:04 AM

Each of the functions above was defined at the value that θ was approaching. However, we have seen that even in the algebraic world, not all functions are undefined at a value, but their limits do exist. The same is true in the trigonometric world.

Evaluating Trigonometric Limits by Rewriting the Function Using Identities

Let's consider for a moment the limit below. Try to evaluate this limit by direct substitution.

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{\sin^2\theta} = \frac{1-\cos 0}{(\sin 0)^2} = \frac{1-1}{0^2} = \frac{0}{0}$$

Again, this function is undefined at $\theta = 0$. However, that does not mean that the limit does not exist. In this case, we can often rewrite the function in terms of a single trig ratio using identities in hopes that the new form of the function is not undefined for the approached value of θ. Do this in the space below.

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{\sin^2\theta} = \lim_{\theta \to 0} \frac{1-\cos\theta}{1-\cos^2\theta} = \lim_{\theta \to 0} \frac{\cancel{1-\cos\theta}}{\cancel{(1-\cos\theta)}(1+\cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{1}{1+\cos\theta} = \frac{1}{1+\cos 0} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

Aug 11-8:03 AM

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Aug 11-8:09 AM

$$\sin^2\theta + \cos^2\theta = 1$$
$$-\cos^2\theta \qquad -\cos^2\theta$$
$$\sin^2\theta = \boxed{1-\cos^2\theta}$$

Aug 11-8:36 AM

1. $\lim\limits_{\theta\to\frac{3\pi}{2}} 3\tan\theta\cos\theta$	2. $\lim\limits_{\theta\to\frac{\pi}{2}} \frac{\sec\theta\cos\theta}{4\theta}$	3. $\lim\limits_{\theta\to\pi} \frac{\cos\theta\tan\theta}{\sin\theta}$

$$\lim_{\theta\to\frac{3\pi}{2}} 3\,\frac{\sin\theta}{\cancel{\cos\theta}}\,\cancel{\cos\theta}$$
$$\lim_{\theta\to\frac{3\pi}{2}} 3\sin\theta$$
$$= 3\sin\left(\frac{3\pi}{2}\right)$$
$$= 3(-1) = \boxed{-3}$$

$$\lim_{\theta\to\frac{\pi}{2}} \frac{\frac{1}{\cancel{\cos\theta}}\,\cancel{\cos\theta}}{4\theta}$$
$$\lim_{\theta\to\frac{\pi}{2}} \frac{1}{4\theta}$$
$$= \frac{1}{4\left(\frac{\pi}{2}\right)} = \boxed{\frac{1}{2\pi}}$$

$$\lim_{\theta\to\pi} \frac{\cancel{\cos\theta}\,\frac{\sin\theta}{\cancel{\cos\theta}}}{\sin\theta}$$
$$\lim_{\theta\to\pi} \frac{\cancel{\sin\theta}}{\cancel{\sin\theta}}$$
$$\lim_{\theta\to\pi} 1 = \boxed{1}$$

Aug 11-8:04 AM