Warm-Up

Simplify the following using Algebra Exponent Rules.

Rule #1: When you are multiplying, you ADD exponents. \((x^3)(x^4) = x^7\)

Rule #2: When you are dividing, you SUBTRACT exponents. \((x^2)/(x^5) = x^{-3} = 1/x^3\)

Rule #3: When you raise an exponent to another exponent, you MULTIPLY exponents. \((x^4)^3 = x^{12}\)

1. \((2s^2t^3)^5\)
2. \(\frac{x^3y}{(x^4)^3} = \frac{x^3y}{x^{12}}\)
3. \((3x^{-2}y^3)^{-3}\)
   \[\frac{3^{-3} x^6 y^{-9}}{3^3 y^{-9}} = \frac{x^6}{27y^9}\]

All missing work and test retakes
Due 9/30/16
Section 11-2 Exponential Functions

Vocabulary

Power Function - functions where the variable is in the base and the number is in the exponent. Ex) \( y=x^5 \)

Exponential Functions - functions where the number is the base and the variable is the exponent

Consider the example \( y=2^x \)

Complete the table and the graph below. Do these values look familiar?

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2^x )</td>
<td>2^(-2) = ( \frac{1}{4} )</td>
<td>2^(-1) = ( \frac{1}{2} )</td>
<td>2^0 = 1</td>
<td>2^1 = 2</td>
<td>2^2 = 4</td>
<td>2^3 = 8</td>
<td>2^4 = 16</td>
<td>2^5 = 32</td>
<td>2^6 = 64</td>
</tr>
</tbody>
</table>

Exponential Growth and Decay

\[ N = N_0 (1 +/- r)^t \]

Where \( N \) is the final amount, \( N_0 \) is the initial amount, \( r \) is the rate of growth/decay, and \( t \) is time. Use + if \( N_0 \) is increasing. Use - if \( N_0 \) is decreasing.

Ex) A car depreciates at a rate of 20% per year. If the car originally cost $20,000, find the value of the car at the end of 2 years.

\[ y = 20000 \left(1 - \frac{2}{10} \right)^2 = 12,800 \]
Ex) In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994? (Don't consider a fractional part of a person.)

\[
y = a \cdot b^x \\
y = 285 \cdot (1+0.75)^9 = 43,872
\]

Ex) One of the most common examples of exponential growth deals with bacteria. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. For example, if we start with only one bacteria which can double every hour, by the end of one day we will have over 16 million bacteria. What would this function look like?

\[
y = 1 \cdot (1+1)\overbrace{2^d} = 16,777,216
\]

Ex) Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

\[
y = 128 \cdot (1-0.5)^5 = 4 \text{ players}
\]

Ex) The pesticide DDT was widely used in the United States until its ban in 1972. DDT is toxic to a wide range of animals and aquatic life, and is suspected to cause cancer in humans. The half-life of DDT can be 15 or more years. Half-life is the amount of time it takes for half of the amount of a substance to decay. Scientists and environmentalists worry about such substances because these hazardous materials continue to be dangerous for many years after their disposal.

For this example, we will set the half-life of the pesticide DDT to be 15 years. If we start with 500 grams of DDT, how much is left after 45 years?

\[
y = 500 \cdot (1-0.5)^3 = 62.5 \text{ grams}
\]
Compound Interest

where $P$ is the principal
$A$ is the final amount
$r$ is the annual interest rate
$n$ is the number of times the interest is compounded
$(\text{annually, semiannually, quarterly, monthly are most common})$
t is the number of years.

Ex) How much money will Hanah have in her account after 7 years if she started with $2500 and the bank offers 5% interest compounded monthly?

$$A = 2500 \left( 1 + \left( \frac{0.05}{12} \right) \right)^{12 \cdot 7} \approx 3545.09$$

Sec 11-3 The Number $e$

$e$ is a number, just like $\pi$ is a number. It is NOT a variable. (You will not be plugging into $e$.)

$$y = e^x$$ is one of the most important exponential growth functions.

Most commonly used for continuous exponential growth/decay.

$$N = N_0 e^{kt}$$

Where $N$ is the final amount, $N_0$ is the initial amount, $k$ is a constant (generally given in the problem) and $t$ is time. (Used with population/science type problems)
When $k$ is positive, the function models exponential growth. When $k$ is negative, the function models exponential decay.

$$A = Pe^{rt}$$ Where $A$ is the final amount, $P$ is the initial amount, $r$ is the annual interest rate, and $t$ is time in years. (Used only when dealing with bank/investment problems)
Compare the balance after 25 years of a $10,000 investment earning 6.75% compounded continuously to the same investment compounded semiannually. Which one earns more money and by how much?

\[ A = Pe^{rt} \]
\[ A = 10000e^{(0.0675 \cdot 25)} = 104059.49 \]

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]
\[ A = 10000 \left(1 + \left(\frac{0.0675}{2}\right)^{2.25}\right) = 52574.62 \]

$30,000 @ 10\text{ years} \quad 9\% \quad \text{continuously}$

\[ A = Pe^{rt} \]
\[ 30000 = Pe^{0.09 \cdot 10} \]
\[ 30000 = 2.457p \]
\[ p = \frac{30000}{2.457} = 12121.0 \]
Logarithmic Functions are the Inverse of Exponential Functions

If \( y = b^x \) then \( x = \log_b y \)

Exponential Form Logarithmic Form
"log base 5 of 25 equals 2"
Ex) Write each equation in exponential form.

a. \( \log_{125} 25 = \frac{2}{3} \)  
   \( 125^{\frac{2}{3}} = 25 \)

b. \( \log_8 2 = \frac{1}{3} \)  
   \( 8^{\frac{1}{3}} = 2 \)

Ex) Write each equation in logarithmic form.

a. \( 4^3 = 64 \)  
   \( \log_4 64 = 3 \)

b. \( 3^{-3} = \frac{1}{27} \)  
   \( \log_3 \frac{1}{27} = -3 \)

Ex) Evaluate the expression

a. \( \log_7 \frac{1}{49} = x \)  
   \( 7^x = \frac{1}{49} \)  
   \( x = -2 \)

b. \( \log_5 625 = x \)  
   \( 5^x = 625 \)  
   \( x = 4 \)

Common Logs: \( \log_{10} \) = Log
Logarithms are used when we are trying to solve for a variable in the exponent.

Ex) How many years will it take $700 invested at 12% compounded annually to amount to $47,000?

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ \frac{47000}{700} = \left(1 + \frac{0.12}{1}\right)^{1\cdot t} \]

\[ 67.143 = 1.12^t \]

\[ \log_{1.12} 67.143 = t \]

\[ \frac{\log(67.143)}{\log(1.12)} = t \]

\[ 37.12 = t \]